UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007

B.Sc / BEd / B.A.S.S II

TITLE OF PAPER : CALCULUS I

COURSE NUMBER : M211

DURATION : THREE (3) HOURS.

INSTRUCTIONS :

1. THIS PAPER CONSISTS OF **SEVEN** QUESTIONS
2. ANSWER **ANY FIVE** QUESTIONS.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

a) Write down a statement (not a proof) of **ROLLE'S** THEOREM. (3)

b) Let $y = f(x)$, $f(x) = \alpha x^2 + \beta x + \phi$, α, β, ϕ being constants, be a parabola. Show that for any $a \in \Re$, $a < b$, the value of c determined in the Mean Value Theorem by $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ is equal to the midpoint of the interval $[a, b]$. (7)

c) Evaluate the following limits

i) $\lim_{x \to 0} \dfrac{(x-3)^2 - 9}{x}$ (3)

ii) $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$ (7)

QUESTION 2

a) Sketch the graph of the following function:

$$f(x) = (x+2)(x-2)^2$$

Indicate all *intercepts, extrema, points of inflection* and *asymptotes* where necessary. (10)

b) Estimate the coordinates of the point where the curve $y = x^3 - x$ crosses the line $y = 1$, using *Newton – Raphson* Method. Take $x_0 = 1$, and maintain 5 decimal places throughout your estimation up to 5 iterations. (10)

QUESTION 3

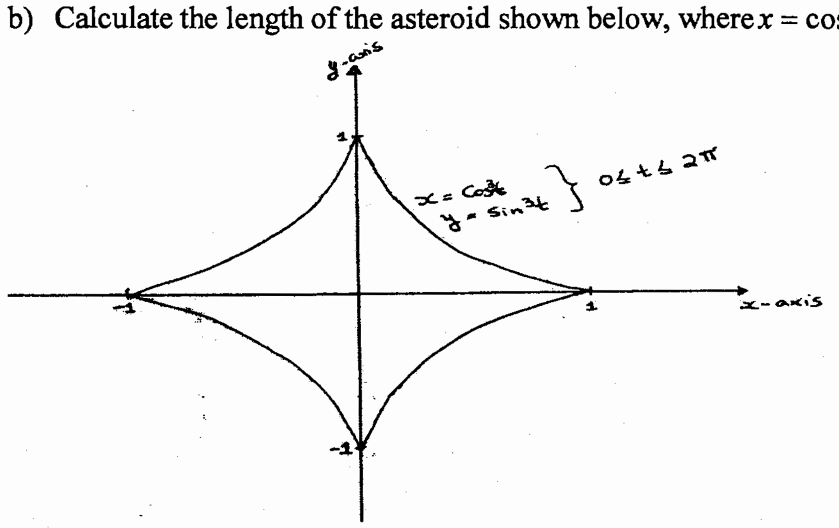
a) ***Show***, using an ***appropriate method***, that if the region between the curve $y=\sqrt{x}$ $\quad 0 \le x \le 4$, and the x - axis , is revolved about the x - axis, the volume of the resulting solid is given by 8π . A sensible sketch of the region referred to here is necessary. (10)

b) The region bounded by the parabola $y=x^2$ and the line $y=2x$ in the first quadrant is rotated about the y- axis to generate a solid. Use the ***washer method*** to determine the volume of the solid. A sensible sketch of the region referred to here is necessary. (10)

QUESTION 4

a) Use the ***shell method*** to **show** that the volume of the solid generated by revolving the region enclosed by the x - axis and the parabola $y=3x-x^2$, about the line $x=-1$, is $\frac{45\pi}{2}$. A sensible sketch of the region referred to here, is necessary. (10)

b) Calculate the length of the asteroid shown below, where $x=\cos^3 t \quad y=\sin^3 t$.



(10)

QUESTION 5

a) Determine if the following sequence converges or diverges. Find the limit if the sequence Converges.

$$a_n = \left(\frac{n+1}{n-1}\right)^n$$

(10)

b) Consider the following sequence.

$$a_n = \frac{n}{n+1}$$

Show that the sequence is

i) Monotonic, and deduce that it is convergent using an appropriate theorem. (4)

ii) Bounded and prove that its *least upper bound* is 1. (6)

QUESTION 6

Discuss the convergence or divergence of each of the following series. State the name of the test used.

a) $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$ (5)

b) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$ (5)

c) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{\sqrt{n}+1}{n+1}\right)$ (5)

d) $\sum_{n=1}^{\infty}\frac{1+\sin n}{n^2}$ (5)

QUESTION 7

a) Use power series to evaluate the following limit

$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$$

Hence or otherwise, show that $\csc x \approx \frac{1}{x}+\frac{x}{6}$ (10)

b) Consider the following power series.

$$\sum_{n=0}^{\infty}\frac{(2x+5)^n}{(n^2+1)3^n}$$

Determine:

i) The centre of convergence, (2)

ii) The radius of convergence, (3)

iii) The interval of convergence of the series. (5)

END OF EXAMINATION