UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2006

BSc. II

TITLE OF PAPER: Mathematics for Scientists

COURSE NUMBER: M215

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:
1. This paper consists of SEVEN questions on TWO pages.
2. Answer ANY FIVE questions.

SPECIAL REQUIREMENTS: NONE.

THIS EXAMINATION PAPER MUST NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) Verify the identity

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$$

for the vectors $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

[8 marks]

(b) Evaluate the following limits:

(i) $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$

(ii) $\lim_{x \to \infty} (x - \sqrt{x^2 + x})$

[12 marks]

Question 2

(a) For the function $f(x) = \sqrt{x - 1}$ in $[1, 3]$, verify that the hypotheses of the Mean Value Theorem are satisfied, and find a number c in $(1, 3)$ whose existence is guaranteed by the theorem.

[10 marks]

(b) Find the first four nonzero terms of the Maclaurin series of the function $f(x) = \cos x$. Hence, deduce the first four nonzero terms of the Maclaurin series of $g(x) = \cos 2x$ and $h(x) = \cos^2 x = \dfrac{1 + \cos 2x}{2}$.

[10 marks]

Question 3

(a) Locate all relative extrema and saddle points of $3x^3 + \frac{3}{2}y^2 - 18xy + 17$.

[10 marks]

(b) Reverse the order of integration and evaluate the resulting integral:

$$\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin y}{y} \, dy dx.$$

[10 marks]

Question 4

(a) Find the general solution of the differential equation

$$(x^3 + y^3)dx + xy^2dy = 0.$$

[10 marks]

(b) Use the method of Lagrange multipliers to find extreme values of $f(x, y) = x - 3y + 1$ subject to $x^2 + 3y^2 = 16$.

[10 marks]

Question 5

(a) Solve the differential equation: $y''' - y'' - 10y' - 8y = 0$.

[10 marks]

(b) Find an approximate value for $\sqrt{35.6} \cdot \sqrt[3]{64.08}$ using differentials.

[10 marks]

Question 6

(a) Solve the following differential equation

$$(2xe^y + e^x)\,dx + (x^2 + 1)e^y\,dy = 0.$$

[10 marks]

(b) Evaluate $\displaystyle\iiint_D \frac{1}{x^2 + y^2 + z^2}\,dV$, where D is the interior of the sphere $x^2 + y^2 + z^2 = 9$.

[10 marks]

Question 7

(a) Use polar coordinates to evaluate the double integral:

$$\iint_R \frac{dx\,dy}{1 + x^2 + y^2}$$

,

where R is the region in the first quadrant bounded by $y = 0$, $y = x$, $x^2+y^2 = 4$.

[10 marks]

(b) Show that if $w = xf\left(\frac{y}{x}\right)$, then $xw_x + yw_y - w = 0$.

[10 marks]

********** *END OF EXAMINATION* ***********