UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2006

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS 2
COURSE NUMBER	:	M 212
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Sketch the curve represented by the parametric equations [11]

$$x = \frac{1}{\sqrt{t+1}}\ , \quad y = \frac{t}{t+1}$$

(b) Change $(r, \theta) = (2, \frac{3\pi}{2})$ from polar to rectangular coordinates. [3]

(c) Change $(x, y) = (1, -1)$ from rectangular to polar coordinates. [3]

(d) Express $(r, \theta, z) = (4, \frac{5\pi}{6}, 3)$ in rectangular coordinates [3]

QUESTION 2

2. (a) Find the length of the curve given by $y = t^2 \quad , \quad x = t^3$ [9]

(b) Sketch the cardiod $r = 2 - 2\ cos\ \theta$. [5]

(c) For $f(x, y) = xe^{x^2 y}$, find $f_x \ \ and \ \ f_y$ and evaluate at (1, ln 2) [6]

QUESTION 3

3. (a) Let $z = xye^{\frac{x}{y}}$ with $x = r\ cos\ \theta \ \ and \ \ y = r\ sin\ \theta$,
find $\frac{dz}{d\theta}$ if $\theta = \frac{\pi}{6}\ , \ r = 2$ [8]

(b) Find $\frac{dy}{dx}$ implicitly if $(x + y)^3 + (x - y)^3 = x^4 + y^4$ [6]

(c) Show that the function $f(x, y) = e^x\ sin\ y$ satisfies $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ [6]

QUESTION 4

4. (a) Define the directional derivative of a function in the direction of vector $\bar{u}$. [3]

(b) Show that if $\bar{u} = cos\theta\ i + sin\theta\ j$, then $D_u f(x, y) = f_x(x, y) cos\theta + f_y(x, y) sin\theta$. [3]

(c) Find the directional derivative if $f(x, y) = 3x^2 - 2y^2$ at $(\frac{-3}{4}, 0)$in the direction from $(\frac{-3}{4}, 0)$ to $(0, 1)$. [4]

(d) Suppose the temperature of a point (x,y,z) in space is given by

$$T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2} \qquad \text{degrees celcius.}$$

(i) In what direction does the temperature increase fastest at the point (1,1,-2) [6]

(ii) What is this maximum rate of increase? [4]

QUESTION 5

5. (a) Sketch the region whose area is represented by the integral [6]

$$\int_0^2 \int_{y^2}^4 dx \; dy$$

(b) Find another iterated integral using the order dy dx to represent the same area and show that both integrals yield the same area. [6]

(c) Find the area of the region R that lies below the parabola $y = 4x - x^2$ and above the line $y = 4 - x$. [8]

QUESTION 6

6. (a) For the surface $z = e^{3y} sin\ 3x$, Find:

(i) the equation of the tangent plane at $P(\frac{\pi}{6}, 0, 2)$. [4]

(ii) the equation of the normal line [4]

(b) Locate all relative extrema and saddle points for [12]

$$f(x, y) = y^2 + 3x^2 - 4x^3 - 12x^2 - 24$$

QUESTION 7

7. (a) Use double polar integral to find the area enclosed by a three petalled rose $r = 3\ sin3\theta$ [11]

(b) Use polar coordinates to evaluate $\int\int (x^2 + y^2) dA$ over a half circle [5]

(c) Evaluate the iterated integral $\int\int\int r cos\theta\ dr\ d\theta\ dz$ over the region enclosed by $0 \leq z \leq 4,\ \ 0 \leq \theta \leq \frac{\pi}{2},\ \ 0 \leq r \leq 2$ [4]