

UNIVERSITY OF SWAZILAND
SUPPLEMENTARY EXAMINATIONS 2006

BSc. / BEd. / B.A.S.S. I

<u>TITLE OF PAPER</u>	:	INTRODUCTION TO CALCULUS
<u>COURSE NUMBER</u>	:	M 115
<u>TIME ALLOWED</u>	:	THREE (3) HOURS
<u>INSTRUCTIONS</u>	:	1. THIS PAPER CONSISTS OF <u>SEVEN</u> QUESTIONS. 2. ANSWER ANY <u>FIVE</u> QUESTIONS
<u>SPECIAL REQUIREMENTS</u>	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) Use the definition to find $f'(x)$

(i) $f(x) = x^2 - 3x + 4$ [5 marks]

(ii) $f(x) = \sqrt{25 - x^2}$ [5 marks]

(b) Use integration by parts to evaluate the following

(i) $\int x^2 \sin x \, dx$ [5 marks]

(ii) $\int x \ln x \, dx$ [5 marks]

Question 2

(a) Find $\frac{dy}{dx}$ for each of the following

(i) $\cos(xy) + x^2y + y = 0$ [5 marks]

(ii) $y = \frac{u^2}{u^2 + 1}$; $u = \sqrt{2x + 1}$ [5 marks]

(b) Find the equation for the tangent to the curve

$$y = x^3 - 3x + 3$$

at the point $(2, 5)$. [5 marks]

(c) Find the slope of the tangent to the curve

$$x^2 + y^2 + xy - 7 = 0$$

at the point $(1, 2)$. [5 marks]

Question 3

(a) Evaluate the following integrals

(i) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(ii) $\int \frac{dx}{(4+x^2)^2}$

[10 marks]

(b) (i) Derive a reduction formula for

$$\int \sec^n x dx$$

(ii) Use the reduction formula in b(i) to evaluate

$$\int \sec^5 x dx$$

[10 marks]

Question 4

Find $\frac{dy}{dx}$ for the following

(a) $y = x^4 - 4x^3 + 6x^2 + 17x + 4$

(b) $y = (x^2 + 3x)^{10}(3x + 7)^{12}$

(c) $y = \frac{2x + 3}{\sqrt{x^2 + 4x}}$

(d) $y = (\sin 4x)^2$

(e) $y = x \tan x$

[20 marks]

Question 5

- (a) Find the area lying above the x -axis and under the parabola

$$y = 4x - x^2.$$

[7 marks]

- (b) Find the derivative $\frac{dy}{dx}$ for

$$y = (x^2 + 1)^x.$$

[7 marks]

- (c) Find the curve whose slope at the point (x, y) is $3x^2$ if the curve is also required to pass through the point $(1, -1)$.

[6 marks]

Question 6

- (a) Find the area bounded by the straight line $4x + y = 2$ and the parabola $y = 2x^2 + 2$.

[6 marks]

- (b) Evaluate the following

(i) $\int \frac{e^x}{(4 + e^x)^2} dx$

[7 marks]

(ii) $\int \frac{1}{\sqrt{2x - x^2}} dx$

[7 marks]

Question 7

(a) Use Leibnitz's rule to find $f^{(4)}(x)$ for

$$f(x) = e^{2x} \sin 3x$$

[8 marks]

(b) Evaluate

$$\int_1^2 (3x - x^2 - 2) dx.$$

[5 marks]

(c) Show that

$$y = \sqrt{x^2 + y^2}$$

satisfies the partial differential equation

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z.$$

[7 marks]

***** END OF EXAMINATION *****