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UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2005

BSc. / BEd. / B.A.S.S. IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M 455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find the acceleration components at a point (1,2,3) at time $t = 1$ for a flow field, $\mathbf{q} = 5x^3\mathbf{i} - 15x^2y\mathbf{j} + t\mathbf{k}$ [6 marks]

   (b) Find the velocity components for a flow represented by the stream function $\psi = 9r^2\sin^2\theta$ [5 marks]

   (c) The following velocity potential corresponds to an inviscid flow around a cylinder of radius a.

$$\phi = U\left(r + \frac{a^2}{r.}\right)\cos\theta + \frac{\theta}{2\pi}$$

   Given that U is a constant,

   i. Find the velocity field. [4 marks]

   ii. Find the stream function. [5 marks]

QUESTION 2

2. (a) The velocity potential for a two-dimensional, irrotational, incompressible flow is $\phi = (x+1)^2 - y^2$.

   i. Find the stream function. [5 marks]

   ii. Find the complex potential $w(z)$, where $z = x + iy$. [5 marks]

   (b) Find the stream function corresponding to the velocity field

$$\mathbf{q} = \frac{y^2 - x^2}{(x^2+y^2)^2}\mathbf{i} + \frac{-2xy}{(x^2+y^2)^2}\mathbf{j}$$

   [10 marks]

QUESTION 3

3. A viscous liquid occupies the space between two coaxial, infinitely-long cylinders. The inner cylinder has radius a and is fixed, while the outer cylinder has radius b and is rotating with constant angular velocity Ω. Let (r, θ, z) be cylindrical polar coordinates with z-axis coinciding with the cylinders' axis such that the outer cylinder is rotating in the direction of increasing θ. Assuming that the velocity of the liquid has the form $\mathbf{q} = u(r)\hat{\theta}$ (where $u(r)$ means that u is a function of r only and $\hat{\theta}$ is a unit vector in the θ direction) and that body forces are negligible,use the Navier-Stokes equations in the form

$$\nabla(\frac{1}{2}\mathbf{q}^2) - \mathbf{q} \times (\nabla \times \mathbf{q}) = -\frac{1}{\rho}\nabla p - \nu \nabla \times (\nabla \times \mathbf{q})$$

to show that

$$u = A(r - a^2/r)$$

where $A = \Omega/(1 - a^2/b^2)$. Also show that

$$p = \frac{\rho A^2 a^2}{2}\left(\frac{r^2}{a^2} - 4\ln\frac{r}{a} - \frac{a^2}{r^2}\right) + C$$

where C is a constant. [20 marks]

QUESTION 4

4. (a) Show that the steady velocity distribution given in cartesian coordinates as

$$\mathbf{q} = (4x - 2z)\mathbf{i} - 5y\mathbf{j} + (z - 2x)\mathbf{k}$$

is a possible motion for an

(i) incompressible fluid [2 marks]

(ii) irrotational fluid [3 marks]

(b) Neglecting any body forces, show that the corresponding pressure field is

$$p = \rho(10zx - 10x^2 - \frac{25}{2}y^2 - \frac{5}{2}z^2) + C$$

where C is a constant. [5 marks]

(c) In the $z = x + iy$ plane, a line vortex of strength $m > 0$, is placed at $z = c$ and another, of strength $-m$, at $z = -c$, where c is a real positive number. Both vortices are held fixed at these locations. Write down the complex potential w for this flow and show that the stream function ψ and the velocity potential ϕ are given by

$$\psi = \frac{m}{4\pi} \log \frac{(x-c)^2 + y^2}{(x+c)^2 + y^2} \quad \text{and} \quad \phi = -\frac{m}{2\pi} \tan^{-1} \frac{2cy}{x^2 + y^2 - c^2}$$

hint: You may set $z - c = r_1 e^{i\theta_1}$ and $z + c = r_2 e^{i\theta_2}$, and use $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$ [10 marks]

QUESTION 5

5. (a) An infinite row of line vortices, each of strength $m > 0$, are placed at the points

$$z = 0, \pm a, \pm 2a, \pm 3a, \ldots, \pm na$$

where a is a real positive number. Use the following infinite product identity

$$\sin\frac{\pi z}{a} = \frac{\pi z}{a}\left(1 - \frac{z}{a}\right)\left(1 + \frac{z}{a}\right)\left(1 - \frac{z}{2a}\right)\left(1 + \frac{z}{2a}\right)\cdots$$

to show that the complex potential for the flow is

$$w(z) = \frac{im}{2\pi}\log\sin\frac{\pi z}{a} \quad + \quad \text{constant}$$

[12 marks]

(b) Find the velocity components and show that

$$(u, v) \to \left(-\frac{m}{2a}, 0\right) \quad \text{as} \quad y \to +\infty$$
$$(u, v) \to \left(+\frac{m}{2a}, 0\right) \quad \text{as} \quad y \to -\infty$$

[8 marks]

QUESTION 6

6. (a) Consider a steady, viscous flow confined between two rigid plates, one at $y = 0$ and the other at $y = h$. Let the lower boundary $y = 0$ be fixed, while the upper boundary $y = h$ is moving in its own plane with velocity $(\bar{U}, 0, 0)$, where $\bar{U}$ is constant. Suppose that the flow is two-dimensional, and hence independent of the z coordinate (i.e $\partial/\partial z = 0$, and the $z-$component of velocity is zero,$w = 0$). Assuming that all $x-$positions are identical so that

there is no change in the x-direction and $\partial/\partial x = 0$. Use the Navier-Stokes equations to show that the velocity profile of the flow is given by

$$\mathbf{q} = \frac{\bar{U}y}{h}$$

[8 marks]

(b) Consider the boundary layer equations in the form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_1\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

Define

$$\xi = \frac{y}{2}\left(\frac{a}{\nu_1 x}\right)^{1/2}, \qquad \psi = -(ax\nu_1)^{1/2}F(\xi),$$

where a and ν_1 are constants, and use the relationships

$$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}$$

to show that equation (1) transforms into the equation

$$FF'' + F''' = 0$$

where the derivatives of F are with respect to ξ. [12 marks]

QUESTION 7

7. Water flows out of a reservoir (see Figure 1) down a pipe of cross sectional area, a. Prove that

(a)

$$h = \left\{ (H + h_0)^{1/2} - \frac{1}{2} t \left[(2ga^2)/(A^2 - a^2) \right]^{1/2} \right\}^2 - H$$

[12 marks]

(b) the time to empty the resevoir is **about**

$$\sqrt{\frac{2}{g}} \left\{ (H + h_0)^{1/2} - H^{1/2} \right\} \frac{A}{a}$$

[8 marks]

where h_0 is the depth of the water at $t = 0$, g is the gravity constant and h and H are the heights shown in Figure 1.

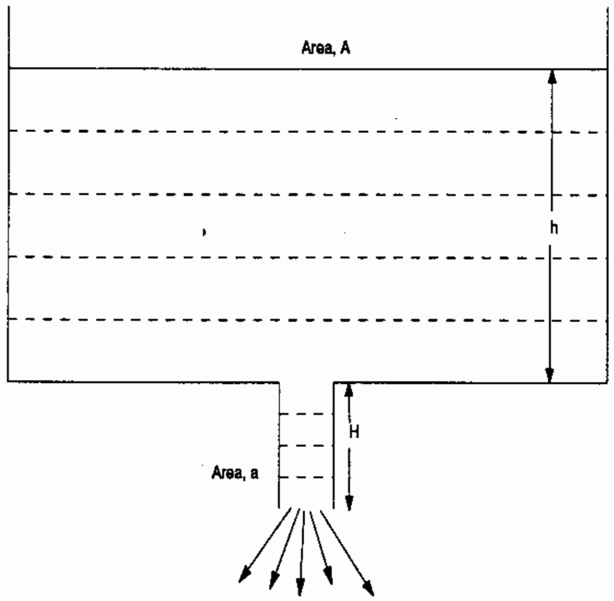

Figure 1: