University of Swaziland

Supplementary Examination 2004/2005

B.Sc./B.Ed./B.A.S.S. III

Title of Paper : **Complex Analysis**

Course Number : M 313

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of **seven questions.**
2. Answer **any five questions**.
3. Your work must be accompanied by appropriate explanations.
4. Use of **cellular phones** during the examination is not allowed.
5. Only non-programmable calculators may be used.

Special requirements: None

The examination paper must not be opened until permission has been granted by the Invigilator.

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Q1.
(a) Find an equation of the circle passing through the points $1-i, 2i, 1+i$
10 [marks]
(b) Given the set A and B represented by $|z-1| < 3$ and $|z-2i| < 2$ respectively, represent geometrically (a) $A \cap B$, and (b) $A \cup B$.
10 [marks]

Q2.
Prove that a necessary and sufficient condition that $w = f(z) = u(x,y) + iv(x,y)$ be analytic in a region R is that the Cauchy-Riemann equations $u_x = v_y, u_y = -v_x$ are satisfied in R, where it is assumed that these partial derivatives are continuous in R.
20 [marks]

Q3.
(a) State and prove the Residue Theorem.
10 [marks]

(b) Use this theorem to evaluate:

$$\int_C \frac{e^z}{(z^2+\pi^2)} dz,$$

where C is the circle $|z| = 4$.
10 [marks]

Q4.
(a) Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$ and determine the region of convergence of this series.

(b) Find the Laurent series about the indicated singularity for each of the following functions: Name the singularity in each case and give the region of convergence of each series.

1. $\dfrac{e^{2z}}{(z-1)^3}; z = 1$

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2. $(z-3)\sin\frac{1}{z+2}; z=-2.$

20 [marks]

Q5.
Describe the poles of the functions:

$$f(z) = \frac{(z+1)}{(z^2+9)} \text{ and } f(z) = \frac{(z^3+2z)}{(z-i)^3}$$

and find the residues at these points.

[20 marks]

Q6
(a) If $f(z)$ is analytic in a simply-connected domain D and if a and z are points in D, show that $F(z) = \int_a^z f(z)dz$ is analytic and $F'(z) = f(z)$.
(b) State Morera's theorem for a simply-connected domain D and prove it.
(c) State Cauchy's inequality and prove it. [You may assume Cauchy's formula for an analytic function and its derivatives.]
(d) State Liouville's theorem and use (c) above or otherwise to prove it.

[20 marks]

Q7.
(a) Describe how Residue theory can be used for evaluating integrals of the form $\int_0^{2\pi} f(\sin x, \cos x)dx$.
(b) Use part (a) to evaluate the integral: $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta}$.

20 [marks]

END OF PAPER