UNIVERSITY OF SWAZILAND

Faculty of Science

Department of Computer Science

Final Examination, December 2009

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the six (6) questions.

Special requirements: The use of electronic calculators is forbidden.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

a) With the aid of a complete truth table, determine whether the following proposition is either tautologous, contradictory or contingent:

$$\neg P \vee Q \wedge R \Rightarrow \neg(Q \wedge \neg P \vee R)$$

[11]

b) By truth table, prove that the following logical equivalence is valid:

$$\neg P \wedge (Q \Rightarrow \neg P) \equiv \neg P$$

[4]

c) By truth table, prove that the entailment law of *Resolution* is valid.

[5]

Question 2

a) Prove the following using the laws of logical equivalence:

$$P \wedge \neg Q \vee \neg Q \wedge P \wedge (R \Leftrightarrow \neg R) \equiv \neg(P \Rightarrow Q)$$

[8]

b) Simplify the following proposition as far as possible, using the laws of logical equivalence:

$$(R \vee S) \wedge (R \Rightarrow S) \vee \neg(Q \wedge (P \Rightarrow P))$$

[12]

Question 3

By natural deduction from the following premises:

- $R \Rightarrow Q$
- $\neg(P \Leftrightarrow Q)$
- $\neg(P \land Q \land \neg R)$

... prove the following conclusions:

a) $P \Rightarrow \neg R$

b) $P \lor Q \lor R$

c) $Q \land R \Rightarrow \neg P$

[20]

Question 4

a) Define the function f(a,b,c) in disjunctive normal form:

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

[5]

b) Define the function f(a,b,c) from part a) in conjunctive normal form.

[8]

c) Implement a circuit for the function g(a,b,c,d) using NOR gates alone:

$$g(a,b,c,d) = a\bar{b}c + \bar{b}d$$

[7]

Question 5

a) Minimize the function f(a,b,c,d) using a Karnaugh map:

$$f(a,b,c,d) =$$

$$ab\bar{c} + bcd + \bar{a}.\bar{b}d + \bar{a}b\bar{c}d$$

Assume that the following inputs are impossible:

$$a\bar{b}.\bar{c}d,\quad a\bar{b}.\bar{d}$$

[9]

b) Minimize the function g(a,b,c,d) using the Quine-McCluskey method:

$$g(a,b,c,d) =$$

$$abcd + abc\bar{d} + ab\bar{c}d + \bar{a}bcd +$$

$$\bar{a}.\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}.\bar{b}.\bar{c}d + \bar{a}.\bar{b}c\bar{d}$$

[11]

Question 6

a) Write the 10-bit binary number 1001011010 in decimal notation.

[2]

b) Write the following decimal numbers in 8-bit binary notation according to the twos-complement system:

i. 101

[2]

ii. -4

[2]

c) Work out the minimum (most negative) integer representable in 6 bits according to the twos-complement system.

[2]

d) Draw a complete circuit diagram of the *half adder*.

[4]

e) Draw a block diagram of a device that inputs two 3-bit binary numbers and outputs their 4-bit sum.

[8]