DEPARTMENT OF CHEMISTRY

UNIVERSITY OF SWAZILAND

C610 – ERM643 **RESEARCH METHODS**

MAY 2012 FINAL EXAMINATION

Time Allowed: Three (3) Hours

Instructions

1. **This examination has five (5) questions and one data sheet. The total number of pages is five (5) including thi page.**

2. **Answer any four questions; diagrams should be clear, large and properly labeled. Marks will be deducted fo improper units and lack of procedural steps in calculations.**

3. **Each question is worth 25 marks.**

Special Requirements

1. **Data sheet.**
2. **Graph Paper.**
3. **Statistical Tables.**
4. **Computer, two multiplug extention cable (4 ports) and USB (4)**

YOU ARE NOT SUPPOSED TO OPEN THIS PAPER UNTIL PERMISSION TO DO SO HAS BEEN GIVE BY THE CHIEF INVIGILATOR.

Question 1 [25]

a) Certified reference materials are useful in the evaluation of reliability and validity of analytical data, especially when the analyte is in a complex matrix. In the determination of copper in sugar cane leaves,

(i) What kind of certified reference materials would be suitable for this analysis? [1]

(ii) How would bulk sampling be carried out to source this material? [3]

(iii) Outline the processes that such a material would undergo during certification. [4]

(iv) Explain how this material would be used to evaluate validity and reliability of copper measurements in sugar cane leaves. [3]

b) Blind samples are useful in analytical quality control in a commercial water laboratory.

(i) What is meant by a blind sample?[1]

(ii) Explain how blind samples are used to evaluate validity and reliability of COD measurements in water. [3]

c) Quality control charts are useful in ensuring that repetitive day to day measurements are under statistical control. An in-house reference material was used to generate the following data over a period of 10 days of measurement of nickel in an ore:

Day #	1	2	3	4	5	6	7	8	9	10
Ni, ppm	103	101	104	99	150	101	110	89	102	100

(i) What is meant by an "in-house reference material"?[1]

(ii) Draw the quality control chart for the nickel determination, assuming that the in-house reference material is 101±4 ppm Ni.[3]

(iii) Which days were the measurements not under statistical control and why?[2]

d) Interlaboratory comparisons are useful in the evaluation of reliability and validity of analytical data. In the measurement of nitrates in a mine pit water sample by ion chromatography, "LAB A" ran ten replicate measurements on the sample, and requested "LAB B" to do the same with the remainder of the sample. The following results were obtained:

LAB A (ppm)	25	23	21	24	25	22	20	22	21	20
LAB B (ppm)	23	29	22	18	15	21	25	29	32	21

(i) Comment on the validity of the results at the 95% confidence level [2]

(ii) Comment on the relative precisions of the two laboratories at the 95% confidence level [2]

QUESTION 2 [25]

(a) (i) Write down the equation that describes the "normal curve of error" in chemometrics, and explain all terms appearing in it.(4)

(ii)Draw the Gaussian curve, and on it indicate the mean and standard deviation (2)

(iii) Under what condition in analytical sampling will the sample variance be the same as the population variance (1)

(b) (i) Differentiate between systematic error and random error in data analysis, and use an example to illustrate this difference (2)

(ii) Differentiate between precision and accuracy in research methods, and use an example to illustrate this difference (2)

(c) The following data was obtained during a spectrophotometric determination of Fe in tap water samples following complexation with bipyridine:

Triplicate absorbance readings for the standards: **1.16 ppm** – 0.120, 0.125, 0.130 ; **2.32 ppm** – 0.248, 0.255, 0.252;

3.48 ppm – 0.382, 0.385, 0.384 ; **4.65 ppm** – 0.504, 0.506, 0.502

Triplicate absorbance readings for the sample are: **0.337, 0.335, 0.340**

(i)Calculate the equation of the calibration curve using the Least Squares Method (5)

(ii) Calculate the absolute error associated with the calibration curve, S_{vc} (3)

(iii) Calculate the absolute error associated with the analytical measurement, S_a (3)

(iv) Calculate the absolute subsampling uncertainty, S_{ss}, in ppm units if five 500-mg portions of the sample were found to contain 3.08ppm, 3.07ppm, 3.11 ppm, 5.01 ppm, and 3.09 ppm. (2)

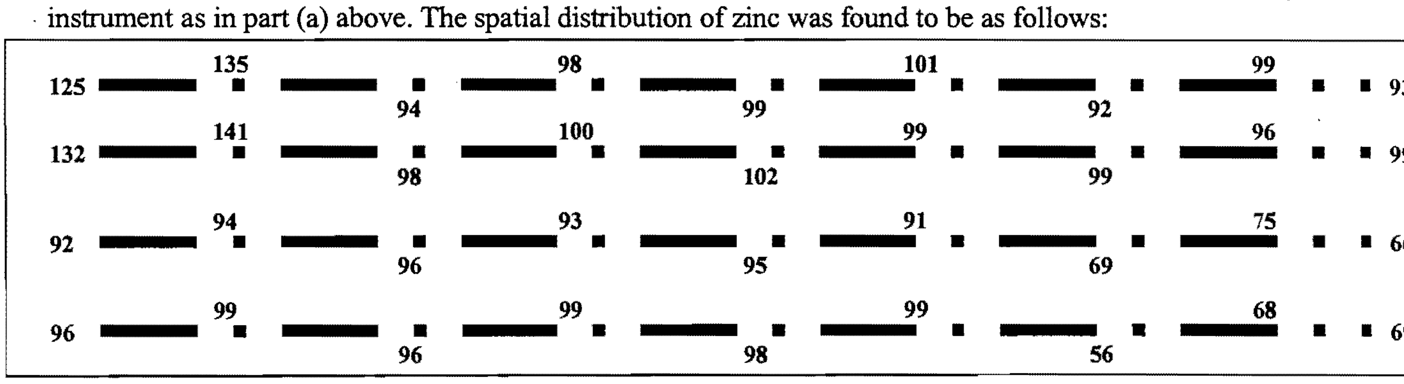
(v) Is the value 5.01 ppm considered part of the data set? Explain why or why not with 90% confidence. (1)

Question 3 [25]

a) Use equations to explain the Benedetti-Pichler approach to sampling of solid samples. What are the short comings of this approach? (4)

b) River sediments present a challenge in their sampling for elemental analysis. What are these challenges, and how are they practically met? (4)

c) Thirty six (36) samples of soil were taken from a field to map the spatial variability of zinc. 500-mg portions of each sample were digested and zinc measured by AA following a standard additions procedure on the same day and same instrument as in part (a) above. The spatial distribution of zinc was found to be as follows:



(i) Use the Kolmogorv-Smirnov test to show that the distribution of zinc in the field is not Gaussian. [6]

(ii) What is meant by a "hot spot" or "coldspot" in analytical sampling? [2]

(iii) In this population, identify and map the points that have resulted in a "hot spot" or "coldspot" causing the non-Gaussian distribution of zinc in the population. [2]

(iv) Calculate the uncertainty due to the sampling operation in ppm units assuming that all other errors are relatively negligible. [2]

(v) Use the Student's t-test equation to determine the minimum number of samples to be taken from the population if the average value of zinc is to be within the error due to sampling at the 95% confidence level. [4]

(vi) If the same samples gave copper results that were twice the zinc uncertainty, how would this have affected the minimum number of samples? (1)

Question 5 [25]

a) Define the term "Principal Component Analysis, PCA". In your brief description include uses, applications, weaknesses and any relevant detail of the technique as applied in chemometrics. [5]

b) Data is sometimes scaled in PCA before application of the techniques. Give reasons. [2]

c) Using the data below calculate: [4]
   i) Eigen values
   ii) Eigen vectors
   iii) Loadings factors
   iv) Score factors

Sample sites	R1	R2	R3	R4	R5
Variables					
Al	12.4	10.39	12.18	12.8	12.6
Fe	10.3	9.01	10.63	9.9	9.5

Show your working. You may use STASTICA to confirm your calculations above.

d) Using the loadings and scores factors show: [6]
   i) Scores plot
   ii) Loadings plot
   iii) Explained (%) variance plot

e) What is the optimum number of principal components, PC's and what is the percentage explained variance as defined by the optimum number of Principal Components? [3]

f) Briefly discuss your findings in your principal component analysis above. In your discussion include comments on sample groups, variable groups, correlations and any observations of vital importance in your findings.[5]

Save all your working from the computer in the USB provided.

Question 6 [25 Marks]

a) Define the term "Cluster Analysis, CA". In your brief description include uses, applications strengths/weaknesses and any relevant details of the technique as applied in chemometrics. [10]

b) Using the data below calculate distance matrix d(i,k). [4]

$$d(i,k) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + ...(x_n - x_{n+1})^2 + (y_n - y_{n+1})^2}$$

Sample sites	R1	R2	R3	R4	R5
Variables					
Ca	13.9	14.1	13.6	14.22	13.00
Na	0.3	0.2	0.2	0.13	0.18

c) Using the average linkage method by Lance and Williams determine the clusters of the data above (in b) and draw the appropriate dendrogram. [5]

Lance and Williams equation states that:

$$d(i1'2,k) = \alpha 1di1,k + \alpha 2di2,k + \beta di1,i2 + \gamma |di1,k - di2,k|$$

where:

$\alpha 1$ is the weight between the distance of first joint object to any other object or cluster

$\alpha 2$ is the weight between the distance of second joint object to any other object or cluster

β is the weight of the distance of both neigbhouring o bjects

γ is the weight of the difference between the distance of neigbhouring objects or clusters.

d) Briefly discuss your findings in your cluster analysis above. In your discussion include comments on clusters, correlations and any observations of vital importance in your findings. [6]

Show all your working. You may use excel/STASTICA to confirm your calculations above.

Save all your working from the computer in the USB provided.

APPENDIX 2

Statistical tables

The following tables are presented for the convenience of the reader, and for use with the simple statistical tests, examples and exercises in this book. They are presented in a format that is compatible with the needs of analytical chemists: the significance level $P = 0.05$ has been used in most cases, and it has been assumed that the number of measurements available is fairly small. Most of these abbreviated tables have been taken, with permission, from *Elementary Statistics Tables* by Henry R. Neave, published by Routledge (Tables A.2–A.4, A.7, A.8, A.11–A.14). The reader requiring statistical data corresponding to significance levels and/or numbers of measurements not covered in the tables is referred to these sources.

Table A.1 $F(z)$, the standard normal cumulative distribution function

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007
−3.2	0.0007	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009
−3.1	0.0010	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013
−3.0	0.0013	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018
−2.9	0.0019	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025
−2.8	0.0026	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034
−2.7	0.0035	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045
−2.6	0.0047	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060
−2.5	0.0062	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080
−2.4	0.0082	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104
−2.3	0.0107	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136
−2.2	0.0139	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174
−2.1	0.0179	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222
−2.0	0.0228	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281

Table A.1 Continued

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.9	0.0287	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351
−1.8	0.0359	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436
−1.7	0.0446	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537
−1.6	0.0548	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655
−1.5	0.0668	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793
−1.4	0.0808	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951
−1.3	0.0968	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131
−1.2	0.1151	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335
−1.1	0.1357	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562
−1.0	0.1587	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814
−0.9	0.1841	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090
−0.8	0.2119	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389
−0.7	0.2420	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709
−0.6	0.2743	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050
−0.5	0.3085	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409
−0.4	0.3446	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783
−0.3	0.3821	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168
−0.2	0.4207	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562
−0.1	0.4602	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6579
0.5	0.6915	0.6950	0.6965	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

Table A.1 Continued

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A.2 The *t*-distribution

Value of t for a confidence interval of	*90%*	*95%*	*98%*	*99%*
Critical value of |t| for P values of number of degrees of freedom	*0.10*	*0.05*	*0.02*	*0.01*
1	6.31	12.71	31.82	63.66
2	2.92	4.30	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.60
5	2.02	2.57	3.36	4.03
6	1.94	2.45	3.14	3.71
7	1.89	2.36	3.00	3.50
8	1.86	2.31	2.90	3.36
9	1.83	2.26	2.82	3.25
10	1.81	2.23	2.76	3.17
12	1.78	2.18	2.68	3.05
14	1.76	2.14	2.62	2.98
16	1.75	2.12	2.58	2.92
18	1.73	2.10	2.55	2.88
20	1.72	2.09	2.53	2.85
30	1.70	2.04	2.46	2.75
50	1.68	2.01	2.40	2.68
∞	1.64	1.96	2.33	2.58

The critical values of $|t|$ are appropriate for a *two*-tailed test. For a *one*-tailed test the value is taken from the column for *twice* the desired *P*-value, e.g. for a one-tailed test, $P = 0.05$, 5 degrees of freedom, the critical value is read from the $P = 0.10$ column and is equal to 2.02.

Table A.3 Critical values of *F* for a one-tailed test ($P = 0.05$)

ν_2	ν_1											
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*12*	*15*
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.745	8.703
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.619
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938
7	5 591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	3.284	3.218
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.845
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.403
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203

ν_1 = number of degrees of freedom of the numerator and ν_2 = number of degrees of freedom of denominator.

ble A.4 Critical values of F for a two-tailed test ($P = 0.05$)

ν_1

1	2	3	4	5	6	7	8	9	10	12	15	20
647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1
38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45
17.44	16.04	15.44	15.10	14.88	14.73	14.62	14:54	14.47	14.42	14.34	14.25	14.17
12.22	10.65	9.979	9.605	9.364	9.197	9.074	8.980	8.905	8.844	8.751	8.657	8.560
10.01	8.434	7.764	7.388	7.146	6.978	6.853	6.757	6.681	6.619	6.525	6.428	6.329
8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461	5.366	5.269	5.168
8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761	4.666	4.568	4.467
7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295	4.200	4.101	3.999
7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964	3.868	3.769	3.667
6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717	3.621	3.522	3.419
6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526	3.430	3.330	3.226
6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374	3.277	3.177	3.073
6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250	3.153	3.053	2.948
6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.209	3.147	3.050	2.949	2.844
6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060	2.963	2.862	2.756
6.115	4.687	4.077	3.729	3.502	3.341	3.219	3.125	3.049	2.986	2.889	2.788	2.681
6.042	4.619	4.011	3.665	3.438	3.277	3.156	3.061	2.985	2.922	2.825	2.723	2.616
5.978	4.560	3.954	3.608	3.382	3.221	3.100	3.005	2.929	2.866	2.769	2.667	2.559
5.922	4.508	3.903	3.559	3.333	3.172	3.051	2.956	2.880	2.817	2.720	2.617	2.509
5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774	2.676	2.573	2.464

number of degrees of freedom of the numerator and ν_2 = number of degrees of freedom of the ıminator.

Table A.5 Critical values of Q ($P = 0.05$) for a two-sided test

Sample size	*Critical value*
4	0.831
5	0.717
6	0.621
7	0.570

Taken from King, E. P. 1958. *J. Am. Statist. Assoc.*, 48: 531.

Table A.6 Critical values of G ($P = 0.05$) for a two-sided test

Sample size	*Critical value*
3	1.155
4	1.481
5	1.715
6	1.887
7	2.020
8	2.126
9	2.215
10	2.290

Taken from *Outliers in Statistical Data*, Vic Barnett and Toby Lewis, 2nd Edition, 1984, John Wiley & Sons Limited.

Table A.7 Critical values of X^2 ($P = 0.05$)

Number of degrees of freedom	*Critical value*
1	3.84
2	5.99
3	7.81
4	9.49
5	11.07
6	12.59
7	14.07
8	15.51
9	16.92
10	18.31

Table A.13 The Spearman rank correlation coefficient. Critical values for ρ at $P = 0.05$

n	*One-tailed test*	*Two-tailed test*
5	0.900	1.000
6	0.829	0.886
7	0.714	0.786
8	0.643	0.738
9	0.600	0.700
10	0.564	0.649
11	0.536	0.618
12	0.504	0.587
13	0.483	0.560
14	0.464	0.538
15	0.446	0.521
16	0.429	0.503
17	0.414	0.488
18	0.401	0.472
19	0.391	0.460
20	0.380	0.447

Table A.14 The Kolmogorov test. Critical two-tailed values for a specified distribution, and for unspecified normal distributions, at $P = 0.05$

n	*Specified distributions*	*Unspecified normal distributions*
3	0.708	0.376
4	0.624	0.375
5	0.563	0.343
6	0.519	0.323
7	0.483	0.304
8	0.454	0.288
9	0.430	0.274
10	0.409	0.262
11	0.391	0.251
12	0.375	0.242
13	0.361	0.234
14	0.349	0.226
15	0.338	0.219
16	0.327	0.213
17	0.318	0.207
18	0.309	0.202
19	0.301	0.197
20	0.294	0.192

The appropriate value is compared with the maximum difference between the experimental and theoretical cumulative frequency curves, as described in the text.

Table A.15 Critical values for C ($P = 0.05$) for $n = 2$

k	*Critical value*
3	0.967
4	0.906
5	0.841
6	0.781
7	0.727
8	0.680
9	0.638
10	0.602

TABLE B Critical values of Student's *t*-distribution

ν \ α	0.9	0.5	0.4	0.2	0.1	0.05	0.02	0.01	0.001	α / ν
1	.158	1.000	1.376	3.078	6.314	12.706	31.821	63.657	636.619	1
2	.142	.816	1.061	1.886	2.920	4.303	6.965	9.925	31.598	2
3	.137	.765	.978	1.638	2.353	3.182	4.541	5.841	12.924	3
4	.134	.741	.941	1.533	2.132	2.776	3.747	4.604	8.610	4
5	.132	.727	.920	1.476	2.015	2.571	3.365	4.032	6.869	5
6	.131	.718	.906	1.440	1.943	2.447	3.143	3.707	5.959	6
7	.130	.711	.896	1.415	1.895	2.365	2.998	3.499	5.408	7
8	.130	.706	.889	1.397	1.860	2.306	2.896	3.355	5.041	8
9	.129	.703	.883	1.383	1.833	2.262	2.821	3.250	4.781	9
10	.129	.700	.879	1.372	1.812	2.228	2.764	3.169	4.587	10
11	.129	.697	.876	1.363	1.796	2.201	2.718	3.106	4.437	11
12	.128	.695	.873	1.356	1.782	2.179	2.681	3.055	4.318	12
13	.128	.694	.870	1.350	1.771	2.160	2.650	3.012	4.221	13
14	.128	.692	.868	1.345	1.761	2.145	2.624	2.977	4.140	14
15	.128	.691	.866	1.341	1.753	2.131	2.602	2.947	4.073	15
16	.128	.690	.865	1.337	1.746	2.120	2.583	2.921	4.015	16
17	.128	.689	.863	1.333	1.740	2.110	2.567	2.898	3.965	17
18	.127	.688	.862	1.330	1.734	2.101	2.552	2.878	3.922	18
19	.127	.688	.861	1.328	1.729	2.093	2.539	2.861	3.883	19
20	.127	.687	.860	1.325	1.725	2.086	2.528	2.845	3.850	20
21	.127	.686	.859	1.323	1.721	2.080	2.518	2.831	3.819	21
22	.127	.686	.858	1.321	1.717	2.074	2.508	2.819	3.792	22
23	.127	.685	.858	1.319	1.714	2.069	2.500	2.807	3.767	23
24	.127	.685	.857	1.318	1.711	2.064	2.492	2.797	3.745	24
25	.127	.684	.856	1.316	1.708	2.060	2.485	2.787	3.725	25
26	.127	.684	.856	1.315	1.706	2.056	2.479	2.779	3.707	26
27	.127	.684	.855	1.314	1.703	2.052	2.473	2.771	3.690	27
28	.127	.683	.855	1.313	1.701	2.048	2.467	2.763	3.674	28
29	.127	.683	.854	1.311	1.699	2.045	2.462	2.756	3.659	29
30	.127	.683	.854	1.310	1.697	2.042	2.457	2.750	3.646	30
40	.126	.681	.851	1.303	1.684	2.021	2.423	2.704	3.551	40
60	.126	.679	.848	1.296	1.671	2.000	2.390	2.660	3.460	60
120	.126	.677	.845	1.289	1.658	1.980	2.358	2.617	3.373	120
∞	.126	.674	.842	1.282	1.645	1.960	2.326	2.576	3.291	∞

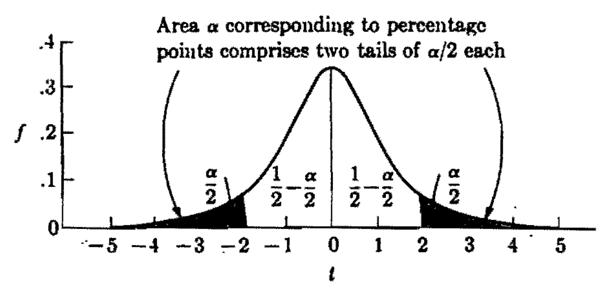


TABLE D Critical values of the chi-square distribution

ν \ α	.995	.975	.9	.5	.1	.05	.025	.01	.005	.001	α / ν
1	0.000	0.000	0.016	0.455	2.706	3.841	5.024	6.635	7.879	10.828	1
2	0.010	0.051	0.211	1.386	4.605	5.991	7.378	9.210	10.597	13.816	2
3	0.072	0.216	0.584	2.366	6.251	7.815	9.348	11.345	12.838	16.266	3
4	0.207	0.484	1.064	3.357	7.779	9.488	11.143	13.277	14.860	18.467	4
5	0.412	0.831	1.610	4.351	9.236	11.070	12.832	15.086	16.750	20.515	5
6	0.676	1.237	2.204	5.348	10.645	12.592	14.449	16.812	18.548	22.458	6
7	0.989	1.690	2.833	6.346	12.017	14.067	16.013	18.475	20.278	24.322	7
8	1.344	2.180	3.490	7.344	13.362	15.507	17.535	20.090	21.955	26.124	8
9	1.735	2.700	4.168	8.343	14.684	16.919	19.023	21.666	23.589	27.877	9
10	2.156	3.247	4.865	9.342	15.987	18.307	20.483	23.209	25.188	29.588	10
11	2.603	3.816	5.578	10.341	17.275	19.675	21.920	24.725	26.757	31.264	11
12	3.074	4.404	6.304	11.340	18.549	21.026	23.337	26.217	28.300	32.910	12
13	3.565	5.009	7.042	12.340	19.812	22.362	24.736	27.688	29.819	34.528	13
14	4.075	5.629	7.790	13.339	21.064	23.685	26.119	29.141	31.319	36.123	14
15	4.601	6.262	8.547	14.339	22.307	24.996	27.488	30.578	32.801	37.697	15
16	5.142	6.908	9.312	15.338	23.542	26.296	28.845	32.000	34.267	39.252	16
17	5.697	7.564	10.085	16.338	24.769	27.587	30.191	33.409	35.718	40.790	17
18	6.265	8.231	10.865	17.338	25.989	28.869	31.526	34.805	37.156	42.312	18
19	6.844	8.907	11.651	18.338	27.204	30.144	32.852	36.191	38.582	43.820	19
20	7.434	9.591	12.443	19.337	28.412	31.410	34.170	37.566	39.997	45.315	20
21	8.034	10.283	13.240	20.337	29.615	32.670	35.479	38.932	41.401	46.797	21
22	8.643	10.982	14.042	21.337	30.813	33.924	36.781	40.289	42.796	48.268	22
23	9.260	11.688	14.848	22.337	32.007	35.172	38.076	41.638	44.181	49.728	23
24	9.886	12.401	15.659	23.337	33.196	36.415	39.364	42.980	45.558	51.179	24
25	10.520	13.120	16.473	24.337	34.382	37.652	40.646	44.314	46.928	52.620	25
26	11.160	13.844	17.292	25.336	35.563	38.885	41.923	45.642	48.290	54.052	26
27	11.808	14.573	18.114	26.336	36.741	40.113	43.194	46.963	49.645	55.476	27
28	12.461	15.308	18.939	27.336	37.916	41.337	44.461	48.278	50.993	56.892	28
29	13.121	16.047	19.768	28.336	39.088	42.557	45.722	49.588	52.336	58.301	29
30	13.787	16.791	20.599	29.336	40.256	43.773	46.979	50.892	53.672	59.703	30
31	14.458	17.539	21.434	30.336	41.422	44.985	48.232	52.191	55.003	61.098	31
32	15.134	18.291	22.271	31.336	42.585	46.194	49.480	53.486	56.329	62.487	32
33	15.815	19.047	23.110	32.336	43.745	47.400	50.725	54.776	57.649	63.870	33
34	16.501	19.806	23.952	33.336	44.903	48.602	51.966	56.061	58.964	65.247	34
35	17.192	20.569	24.797	34.336	46.059	49.802	53.203	57.342	60.275	66.619	35
36	17.887	21.336	25.643	35.336	47.212	50.998	54.437	58.619	61.582	67.985	36
37	18.586	22.106	26.492	36.335	48.363	52.192	55.668	59.892	62.884	69.346	37
38	19.289	22.878	27.343	37.335	49.513	53.384	56.896	61.162	64.182	70.703	38
39	19.996	23.654	28.196	38.335	50.660	54.572	58.120	62.428	65.476	72.055	39
40	20.707	24.433	29.051	39.335	51.805	55.758	59.342	63.691	66.766	73.402	40
41	21.421	25.215	29.907	40.335	52.949	56.942	60.561	64.950	68.053	74.745	41
42	22.138	25.999	30.765	41.335	54.090	58.124	61.777	66.206	69.336	76.084	42
43	22.859	26.785	31.625	42.335	55.230	59.304	62.990	67.459	70.616	77.419	43
44	23.584	27.575	32.487	43.335	56.369	60.481	64.202	68.710	71.893	78.750	44
45	24.311	28.366	33.350	44.335	57.505	61.656	65.410	69.957	73.166	80.077	45
46	25.042	29.160	34.215	45.335	58.641	62.830	66.617	71.201	74.437	81.400	46
47	25.775	29.956	35.081	46.335	59.774	64.001	67.821	72.443	75.704	82.720	47
48	26.511	30.755	35.949	47.335	60.907	65.171	69.023	73.683	76.969	84.037	48
49	27.249	31.555	36.818	48.335	62.038	66.339	70.222	74.919	78.231	85.351	49
50	27.991	32.357	37.689	49.335	63.167	67.505	71.420	76.154	79.490	86.661	50

TABLE D Crit

ν \ α	.995	.97
51	28.735	33.16
52	29.481	33.96
53	30.230	34.77
54	30.981	35.58
55	31.735	36.39
56	32.490	37.212
57	33.248	38.027
58	34.008	38.844
59	34.770	39.662
60	35.534	40.482
61	36.300	41.303
62	37.068	42.126
63	37.838	42.950
64	38.610	43.776
65	39.383	44.603
66	40.158	45.431
67	40.935	46.261
68	41.713	47.092
69	42.494	47.924
70	43.275	48.758
71	44.058	49.592
72	44.843	50.428
73	45.629	51.265
74	46.417	52.103
75	47.206	52.942
76	47.997	53.782
77	48.788	54.623
78	49.582	55.466
79	50.376	56.309
80	51.172	57.153
81	51.969	57.998
82	52.767	58.845
83	53.567	59.692
84	54.368	60.540
85	55.170	61.389
86	55.973	62.239
87	56.777	63.089
88	57.582	63.941
89	58.389	64.793
90	59.196	65.647
91	60.005	66.501
92	60.815	67.356
93	61.625	68.211
94	62.437	69.068
95	63.250	69.925
96	64.063	70.783
97	64.878	71.642
98	65.694	72.501
99	66.510	73.361
100	67.328	74.222

ution

1	.005	.001	α/ν
35	7.879	10.828	1
10	10.597	13.816	2
45	12.838	16.266	3
77	14.860	18.467	4
86	16.750	20.515	5
12	18.548	22.458	6
75	20.278	24.322	7
90	21.955	26.124	8
66	23.589	27.877	9
09	25.188	29.588	10
25	26.757	31.264	11
17	28.300	32.910	12
88	29.819	34.528	13
41	31.319	36.123	14
78	32.801	37.697	15
00	34.267	39.252	16
09	35.718	40.790	17
05	37.156	42.312	18
91	38.582	43.820	19
56	39.997	45.315	20
32	41.401	46.797	21
39	42.796	48.268	22
38	44.181	49.728	23
30	45.558	51.179	24
14	46.928	52.620	25
12	48.290	54.052	26
53	49.645	55.476	27
78	50.993	56.892	28
38	52.336	58.301	29
02	53.672	59.703	30
01	55.003	61.098	31
36	56.329	62.487	32
76	57.649	63.870	33
51	58.964	65.247	34
12	60.275	66.619	35
19	61.582	67.985	36
02	62.884	69.346	37
52	64.182	70.703	38
28	65.476	72.055	39
01	66.766	73.402	40
50	68.053	74.745	41
06	69.336	76.084	42
59	70.616	77.419	43
10	71.893	78.750	44
57	73.166	80.077	45
01	74.437	81.400	46
13	75.704	82.720	47
33	76.969	84.037	48
9	78.231	85.351	49
54	79.490	86.661	50

TABLE D Critical values of the chi-square distribution (*continued*)

ν\α	.995	.975	.9	.5	.1	.05	.025	.01	.005	.001	α/ν
51	28.735	33.162	38.560	50.335	64.295	68.669	72.616	77.386	80.747	87.968	51
52	29.481	33.968	39.433	51.335	65.422	69.832	73.810	78.616	82.001	89.272	52
53	30.230	34.776	40.308	52.335	66.548	70.993	75.002	79.843	83.253	90.573	53
54	30.981	35.586	41.183	53.335	67.673	72.153	76.192	81.069	84.502	91.872	54
55	31.735	36.398	42.060	54.335	68.796	73.311	77.380	82.292	85.749	93.168	55
56	32.490	37.212	42.937	55.335	69.918	74.468	78.567	83.513	86.994	94.460	56
57	33.248	38.027	43.816	56.335	71.040	75.624	79.752	84.733	88.237	95.751	57
58	34.008	38.844	44.696	57.335	72.160	76.778	80.936	85.950	89.477	97.039	58
59	34.770	39.662	45.577	58.335	73.279	77.931	82.117	87.166	90.715	98.324	59
60	35.534	40.482	46.459	59.335	74.397	79.082	83.298	88.379	91.952	99.607	60
61	36.300	41.303	47.342	60.335	75.514	80.232	84.476	89.591	93.186	100.888	61
62	37.068	42.126	48.226	61.335	76.630	81.381	85.654	90.802	94.419	102.166	62
63	37.838	42.950	49.111	62.335	77.745	82.529	86.830	92.010	95.649	103.442	63
64	38.610	43.776	49.996	63.335	78.860	83.675	88.004	93.217	96.878	104.716	64
65	39.383	44.603	50.883	64.335	79.973	84.821	89.177	94.422	98.105	105.988	65
66	40.158	45.431	51.770	65.335	81.085	85.965	90.349	95.626	99.331	107.258	66
67	40.935	46.261	52.659	66.335	82.197	87.108	91.519	96.828	100.55	108.526	67
68	41.713	47.092	53.548	67.334	83.308	88.250	92.689	98.028	101.78	109.791	68
69	42.494	47.924	54.438	68.334	84.418	89.391	93.856	99.228	103.00	111.055	69
70	43.275	48.758	55.329	69.334	85.527	90.531	95.023	100.43	104.21	112.317	70
71	44.058	49.592	56.221	70.334	86.635	91.670	96.189	101.62	105.43	113.577	71
72	44.843	50.428	57.113	71.334	87.743	92.808	97.353	102.82	106.65	114.835	72
73	45.629	51.265	58.006	72.334	88.850	93.945	98.516	104.01	107.86	116.092	73
74	46.417	52.103	58.900	73.334	89.956	95.081	99.678	105.20	109.07	117.346	74
75	47.206	52.942	59.795	74.334	91.061	96.217	100.84	106.39	110.29	118.599	75
76	47.997	53.782	60.690	75.334	92.166	97.351	102.00	107.58	111.50	119.850	76
77	48.788	54.623	61.586	76.334	93.270	98.484	103.16	108.77	112.70	121.100	77
78	49.582	55.466	62.483	77.334	94.373	99.617	104.32	109.96	113.91	122.348	78
79	50.376	56.309	63.380	78.334	95.476	100.75	105.47	111.14	115.12	123.594	79
80	51.172	57.153	64.278	79.334	96.578	101.88	106.63	112.33	116.32	124.839	80
81	51.969	57.998	65.176	80.334	97.680	103.01	107.78	113.51	117.52	126.082	81
82	52.767	58.845	66.076	81.334	98.780	104.14	108.94	114.69	118.73	127.324	82
83	53.567	59.692	66.976	82.334	99.880	105.27	110.09	115.88	119.93	128.565	83
84	54.368	60.540	67.876	83.334	100.98	106.39	111.24	117.06	121.13	129.804	84
85	55.170	61.389	68.777	84.334	102.08	107.52	112.39	118.24	122.32	131.041	85
86	55.973	62.239	69.679	85.334	103.18	108.65	113.54	119.41	123.52	132.277	86
87	56.777	63.089	70.581	86.334	104.28	109.77	114.69	120.59	124.72	133.512	87
88	57.582	63.941	71.484	87.334	105.37	110.90	115.84	121.77	125.91	134.745	88
89	58.389	64.793	72.387	88.334	106.47	112.02	116.99	122.94	127.11	135.978	89
90	59.196	65.647	73.291	89.334	107.56	113.15	118.14	124.12	128.30	137.208	90
91	60.005	66.501	74.196	90.334	108.66	114.27	119.28	125.29	129.49	138.438	91
92	60.815	67.356	75.101	91.334	109.76	115.39	120.43	126.46	130.68	139.666	92
93	61.625	68.211	76.006	92.334	110.85	116.51	121.57	127.63	131.87	140.893	93
94	62.437	69.068	76.912	93.334	111.94	117.63	122.72	128.80	133.06	142.119	94
95	63.250	69.925	77.818	94.334	113.04	118.75	123.86	129.97	134.25	143.344	95
96	64.063	70.783	78.725	95.334	114.13	119.87	125.00	131.14	135.43	144.567	96
97	64.878	71.642	79.633	96.334	115.22	120.99	126.14	132.31	136.62	145.789	97
98	65.694	72.501	80.541	97.334	116.32	122.11	127.28	133.48	137.80	147.010	98
99	66.510	73.361	81.449	98.334	117.41	123.23	128.42	134.64	138.99	148.230	99
100	67.328	74.222	82.358	99.334	118.50	124.34	129.56	135.81	140.17	149.449	100

TABLE X Critical values of the δ-corrected one-sample Kolmogorov–Smirnov statistic

		α				
n	δ	**0.2**	**0.1**	**0.05**	**0.02**	**0.01**
3	0.0	.35477	.41811	.46702	.53456	.57900
	0.5	.39814	.46938	.54093	.61789	.66234
	1.0	.53584	.63160	.70760	.78456	.82900
4	0.0	.33435	.39075	.44641	.50495	.54210
	0.5	.36765	.44022	.49894	.56387	.60924
	1.0	.46154	.53829	.60468	.68377	.73409
5	0.0	.31556	.37359	.42174	.47692	.51576
	0.5	.34698	.40945	.46328	.52718	.56853
	1.0	.41172	.48153	.54273	.61133	.65692
6	0.0	.30244	.35522	.40045	.45440	.48988
	0.5	.32704	.38466	.43593	.49407	.53327
	1.0	.37706	.44074	.49569	.55969	.60287
7	0.0	.28991	.33905	.38294	.43337	.46761
	0.5	.31005	.36464	.41200	.46701	.50438
	1.0	.35066	.40892	.46010	.51968	.55970
8	0.0	.27828	.32538	.36697	.41522	.44819
	0.5	.29581	.34712	.39177	.44404	.47929
	1.0	.32925	.38365	.43160	.48732	.52519
9	0.0	.26794	.31325	.35277	.39922	.43071
	0.5	.28355	.33191	.37446	.42404	.45776
	1.0	.31157	.36287	.40794	.46067	.49652
10	0.0	.25884	.30221	.34022	.38481	.41517
	0.5	.27260	.31866	.35925	.40662	.43893
	1.0	.29668	.34525	.38798	.43809	.47220
11	0.0	.25071	.29227	.32894	.37187	.40122
	0.5	.26284	.30697	.34577	.39125	.42225
	1.0	.28388	.33008	.37084	.41864	.45127
12	0.0	.24325	.28330	.31869	.36019	.38856
	0.5	.25410	.29648	.33376	.37751	.40738
	1.0	.27269	.31686	.35588	.40167	.43298
13	0.0	.23639	.27515	.30935	.34954	.37703
	0.5	.24624	.28703	.32297	.36516	.39401
	1.0	.26279	.30520	.34265	.38668	.41680
14	0.0	.23010	.26767	.30081	.33980	.36649
	0.5	.23909	.27846	.31319	.35398	.38190
	1.0	.25395	.29478	.33086	.37331	.40238
15	0.0	.22430	.26077	.29296	.33083	.35679
	0.5	.23255	.27064	.30426	.34379	.37087
	1.0	.24600	.28541	.32026	.36128	.38940
16	0.0	.21895	.25439	.28570	.32256	.34784
	0.5	.22653	.26347	.29608	.33446	.36076
	1.0	.23879	.27692	.31065	.35039	.37764
17	0.0	.21397	.24847	.27897	.31489	.33953
	0.5	.22098	.25686	.28855	.32586	.35145
	1.0	.23221	.26918	.30189	.34045	.36691
18	0.0	.20933	.24296	.27270	.30775	.33181
	0.5	.21582	.25073	.28158	.31792	.34284
	1.0	.22617	.26208	.29386	.33134	.35707
19	0.0	.20498	.23781	.26685	.30108	.32459
	0.5	.21103	.24504	.27511	.31054	.33485
	1.0	.22060	.25553	.28646	.32295	.34801
20	0.0	.20089	.23298	.26137	.29484	.31784
	0.5	.20656	.23973	.26908	.30366	.32741
	1.0	.21544	.24947	.27961	.31518	.33962
21	0.0	.19705	.22844	.25622	.28898	.31149
	0.5	.20236	.23477	.26343	.29723	.32045
	1.0	.21064	.24384	.27325	.30796	.33182

TABLE X Critical values of the δ-corrected one-sample Kolmogorov–Smirnov statistic (*continued*)

		α				
n	δ	0.2	0.1	0.05	0.02	0.01
22	0.0	.19343	.22416	.25136	.28346	.30552
	0.5	.19843	.23011	.25814	.29121	.31393
	1.0	.20616	.23859	.26732	.30123	.32456
23	0.0	.19001	.22012	.24679	.27825	.29989
	0.5	.19472	.22572	.25317	.28554	.30780
	1.0	.20197	.23367	.26176	.29494	.31776
24	0.0	.18677	.21630	.24245	.27333	.29456
	0.5	.19121	.22159	.24847	.28021	.30202
	1.0	.19804	.22906	.25656	.28904	.31138
25	0.0	.18370	.21268	.23835	.26866	.28951
	0.5	.18790	.21768	.24404	.27516	.29657
	1.0	.19433	.22472	.25166	.28349	.30539
26	0.0	.18077	.20924	.23445	.26423	.28472
	0.5	.18476	.21397	.23984	.27039	.29140
	1.0	.19084	.22063	.24704	.27825	.29973
27	0.0	.17799	.20596	.23074	.26001	.28016
	0.5	.18178	.21046	.23586	.26586	.28650
	1.0	.18753	.21676	.24267	.27330	.29439
28	0.0	.17533	.20283	.22721	.25600	.27582
	0.5	.17894	.20712	.23208	.26156	.28185
	1.0	.18440	.21309	.23853	.26861	.28933
29	0.0	.17280	.19985	.22383	.25217	.27168
	0.5	.17624	.20393	.22847	.25747	.27742
	1.0	.18142	.20961	.23461	.26417	.28452
30	0.0	.17037	.19700	.22061	.24851	.26772
	0.5	.17365	.20090	.22504	.25356	.27320
	1.0	.17859	.20630	.23088	.25994	.27996
31	0.0	.16805	.19427	.21752	.24501	.26393
	0.5	.17119	.19800	.22176	.24983	.26917
	1.0	.17589	.20314	.22732	.25591	.27561
32	0.0	.16582	.19166	.21457	.24165	.26030
	0.5	.16882	.19522	.21862	.24627	.26531
	1.0	.17332	.20014	.22393	.25207	.27146
33	0.0	.16368	.18915	.21173	.23843	.25683
	0.5	.16656	.19256	.21561	.24286	.26162
	1.0	.17086	.19726	.22069	.24840	.26750
34	0.0	.16162	.18674	.20901	.23534	.25348
	0.5	.16439	.19001	.21273	.23958	.25808
	1.0	.16850	.19451	.21759	.24490	.26371
35	0.0	.15964	.18442	.20639	.23237	.25027
	0.5	.16230	.18756	.20996	.23644	.25469
	1.0	.16625	.19188	.21462	.24154	.26008
36	0.0	.15774	.18218	.20387	.22951	.24718
	0.5	.16029	.18521	.20730	.23343	.25143
	1.0	.16408	.18935	.21178	.23831	.25660
37	0.0	.15590	.18003	.20144	.22676	.24421
	0.5	.15836	.18294	.20474	.23052	.24829
	1.0	.16200	.18692	.20904	.23522	.25326
38	0.0	.15413	.17796	.19910	.22410	.24134
	0.5	.15650	.18076	.20228	.22773	.24527
	1.0	.16000	.18459	.20642	.23225	.25005
39	0.0	.15242	.17595	.19684	.22154	.23857
	0.5	.15471	.17866	.19991	.22504	.24236
	1.0	.15808	.18234	.20389	.22938	.24696
40	0.0	.15076	.17402	.19465	.21907	.23589
	0.5	.15297	.17663	.19762	.22244	.23955
	1.0	.15622	.18018	.20145	.22663	.24399