

**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION 2011/12**

**TITLE OF PAPER: PHYSICAL CHEMISTRY**

**COURSE NUMBER: C302**

**TIME: THREE (3) HOURS**

**INSTRUCTIONS:**

There are **six** questions. Each question is worth 25 marks. Answer **any four** questions.

A list of integrals, a data sheet and a periodic table are attached

Non-programmable electronic calculators may be used.

**DO NOT OPEN THIS PAPER UNTIL PERMISSION TO DO SO HAS BEEN GRANTED BY THE CHIEF INVIGILATOR.**

**Question 1 (25 marks)**

- (a) For a monatomic gas, one measure of the average speed of the atoms is the root mean square speed,  $v_{\text{rms}} = \langle v^2 \rangle^{1/2} = (3kT/m)^{1/2}$ , in which  $m$  is the mass of the gas atom and  $k$  the Boltzmann constant. Use this formula to calculate the de Broglie wavelength for xenon atoms at 100 K and 500 K. [6]
- (b) The following data were observed in an experiment on the photoelectric effect from potassium:

Kinetic energy $\times 10^{19}$ J	4.49	3.09	1.89	1.34	0.700	0.311
Wavelength nm	250	300	350	400	450	500

- (i) Plot a graph of kinetic energy against frequency in  $\text{s}^{-1}$ .
- (ii) Use your graph to determine the value of Planck's constant, the work-function and threshold frequency of potassium. [8]
- (c) Which of the following functions are eigenfunctions of  $\frac{d^2}{dx^2}$ ? For each eigenfunction give the eigenvalue:
- (i)  $5\sin 3x$       (ii)  $5x^3$       (iii)  $5e^{-3x}$       (iv)  $\ln x$  [6]
- (d) Normalize the function  $\psi = \cos\theta$ ,  $0 \leq \theta \leq 2\pi$  [5]

**Question 2 (25 marks)**

- (a) Consider the following species:  $N_2^+$ ,  $N_2$ ,  $N_2^-$ ,  $N_2^{2-}$
- (i) Sketch the molecular orbital energy diagram for  $N_2$ . [2]
- (ii) Write the electron configuration for each of the four species listed above and calculate the bond order for each. [8]
- (iii) Arrange the species in order of increasing bond energy [1]
- (iv) Arrange the species in order of increasing bond length [1]
- (v) Derive the ground state term symbol for each species [8]
- (b) Distinguish between bonding and anti-bonding molecular orbitals [5]

**Question 3 (25 marks)**

- (a) Determine the number of translational, rotational and vibrational degrees of freedom in the following molecules:  
(i) CH<sub>3</sub>Cl                      (ii) OCS                      (iii) C<sub>6</sub>H<sub>6</sub>                      (iv) H<sub>2</sub>CO                      [6]
- (b) Classify each of the following molecules as a spherical, a symmetric or an asymmetric top:  
(i) CH<sub>3</sub>Cl                      (ii) CCl<sub>4</sub>                      (iii) SO<sub>2</sub>                      (iv) SF<sub>6</sub>                      [4]
- (c) The rotational constant of <sup>2</sup>D<sup>19</sup>F determined from microwave spectroscopy is 11.007 cm<sup>-1</sup>. The atomic masses of <sup>19</sup>F and <sup>2</sup>D are 18.9984032 u and 2.0141018 u, respectively. Calculate the bond length in <sup>2</sup>D<sup>19</sup>F to the maximum number of significant figures consistent with this information. [7]
- (d) The pure rotational Raman spectrum of <sup>14</sup>N<sub>2</sub> shows a spacing of 7.99 cm<sup>-1</sup> between adjacent rotational lines.
- (i) Calculate the value of the rotational constant B. [2]
- (ii) What is the spacing between the unshifted line  $\nu_{ex}$  and the pure rotational line closest to  $\nu_{ex}$ ? [2]
- (iii) If 540.8 nm radiation from an argon laser is used as the exciting radiation, find the wavelength of the two pure rotational Raman lines nearest the unshifted lines. [4]

**Question 4(25 marks)**

- (a) Describe the fundamental vibrational modes of H<sub>2</sub>O and CO<sub>2</sub>. For each molecule indicate which modes will show infrared activity and why. [8]
- (b) Explain the difference between a “hot band” and an “overtone band” in infrared spectra. How would you distinguish the two experimentally? [5]
- (c) The anharmonicity constant for <sup>35</sup>Cl<sup>19</sup>F is 1.25 x 10<sup>-2</sup> and the fundamental frequency is 793.2 cm<sup>-1</sup>. The isotopic masses for <sup>35</sup>Cl and <sup>19</sup>F are 34.9688 u and 18.9984 u, respectively.
- (i) Calculate the energies of the first four vibrational levels in cm<sup>-1</sup>. [4]
- (ii) Calculate the difference in energy between the  $v = 25$  and  $v = 26$  levels using (1) the harmonic oscillator model and (2) the anharmonic oscillator model. Comment on the difference of your results from the two calculations. [4]
- (iii) Calculate the force constant of the bond in this molecule. [4]

**Question 5(25 marks)**

(a) Describe the physical origin of quantization energy for a particle confined to moving inside a one-dimensional box. [5]

(b) The ground state normalized wavefunction of a particle in a one-dimensional box of

length L is: 
$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

(i) Show that  $\psi$  is an eigenfunction of the operator below and give the eigenvalue.

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \quad [5]$$

(ii) Find the average value of the coordinate  $x$ . [5]

(iii) Find the average value of the linear momentum  $p_x$ . [5]

(iv) What is the probability of finding the particle in the middle third of the box? [5]

**Question 6(25marks)**

(a) The term symbol of a particular state is quoted as  ${}^4D_{5/2}$ . What are the values of L, S and J? What is the minimum number of electrons which could give rise to this? Suggest a possible electronic configuration. [4]

(b) Derive the term symbols for the electron configuration  $ns^1nd^1$  [4]

(c) The term symbol of a particular states of two different atoms are quoted as (i)  ${}^2D_{7/2}$  and (ii)  ${}^0P_1$ . Explain why these are erroneous. [4]

(d) What is spin orbit coupling? [3]

(e) Which of the following transitions are allowed and which are forbidden in a hydrogenic atom. Explain.

(i)  $2p \rightarrow 3p$       (ii)  $2p \rightarrow 5s$       (iii)  $3d \rightarrow 3s$  [6]

(f) State whether the following transitions are allowed or forbidden in the emission spectrum of helium. In each case give a reason for your answer.

(i)  $4\ {}^3P_2 \rightarrow 2\ {}^3S_1$       (ii)  $4\ {}^1D_2 \rightarrow 2\ {}^1S_0$  [4]

### USEFUL INTEGRALS

$$(1) \quad \int x^n dx = \frac{1}{(n+1)}x^{n+1}, \quad n \neq -1$$

$$(2) \quad \int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \quad a > 0, \quad n \text{ positive integer}$$

$$(3) \quad \int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a} \sin 2ax + \text{constant}$$

$$(4) \quad \int \sin \theta d\theta = -\cos \theta + \text{constant}$$

$$(5) \quad \int x \sin^2 ax dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a} + \text{constant}$$

$$(6) \quad \int \cos^2 \theta d\theta = \frac{\theta}{2} + \frac{1}{4} \sin 2\theta + \text{constant}$$

$$(7) \quad \int_0^{\pi} x \sin x dx = \frac{\pi^2}{2}$$

$$(8) \quad d\tau = r^2 dr \sin \theta d\theta \, d\phi$$

## General data and fundamental constants

Quantity	Symbol	Value
Speed of light	$c$	$2.997\ 924\ 58 \times 10^8 \text{ m s}^{-1}$
Elementary charge	$e$	$1.602\ 177 \times 10^{-19} \text{ C}$
Faraday constant	$F = N_A e$	$9.6485 \times 10^4 \text{ C mol}^{-1}$
Boltzmann constant	$k$	$1.380\ 66 \times 10^{-23} \text{ J K}^{-1}$
Gas constant	$R = N_A k$	$8.314\ 51 \text{ J K}^{-1} \text{ mol}^{-1}$ $8.205\ 78 \times 10^{-2} \text{ dm}^3 \text{ atm K}^{-1} \text{ mol}^{-1}$ $6.2364 \times 10 \text{ L Torr K}^{-1} \text{ mol}^{-1}$
Planck constant	$h$ $\hbar = h/2\pi$	$6.626\ 08 \times 10^{-34} \text{ J s}$ $1.054\ 57 \times 10^{-34} \text{ J s}$
Avogadro constant	$N_A$	$6.022\ 14 \times 10^{23} \text{ mol}^{-1}$
Atomic mass unit	$u$	$1.660\ 54 \times 10^{-27} \text{ Kg}$
Mass		
electron	$m_e$	$9.109\ 39 \times 10^{-31} \text{ Kg}$
proton	$m_p$	$1.672\ 62 \times 10^{-27} \text{ Kg}$
neutron	$m_n$	$1.674\ 93 \times 10^{-27} \text{ Kg}$
Vacuum permittivity	$\epsilon_0 = 1/c^2 \mu_0$ $4\pi\epsilon_0$	$8.854\ 19 \times 10^{-12} \text{ J}^{-1} \text{ C}^2 \text{ m}^{-1}$ $1.112\ 65 \times 10^{-10} \text{ J}^{-1} \text{ C}^2 \text{ m}^{-1}$
Vacuum permeability	$\mu_0$	$4\pi \times 10^{-7} \text{ J s}^2 \text{ C}^{-2} \text{ m}^{-1}$ $4\pi \times 10^{-7} \text{ T}^2 \text{ J}^{-1} \text{ m}^3$
Magneton		
Bohr	$\mu_B = e\hbar/2m_e$	$9.274\ 02 \times 10^{-24} \text{ J T}^{-1}$
nuclear	$\mu_N = e\hbar/2m_p$	$5.050\ 79 \times 10^{-27} \text{ J T}^{-1}$
g value	$g_e$	2.002 32
Bohr radius	$a_0 = 4\pi\epsilon_0 \hbar^2 / m_e e^2$	$5.291\ 77 \times 10^{-11} \text{ m}$
Fine-structure constant	$\alpha = \mu_0 e^2 c / 2\hbar$	$7.297\ 35 \times 10^{-3}$
Rydberg constant	$R_\infty = m_e e^4 / 8\hbar^3 c \epsilon_0^2$	$1.097\ 37 \times 10^7 \text{ m}^{-1}$
Standard acceleration of free fall	$g$	$9.806\ 65 \text{ m s}^{-2}$
Gravitational constant	$G$	$6.672\ 59 \times 10^{-11} \text{ N m}^2 \text{ Kg}^{-2}$

## Conversion factors

1 cal	=	4.184 joules (J)	1 erg	=	$1 \times 10^{-7} \text{ J}$
1 eV	=	$1.602\ 2 \times 10^{-19} \text{ J}$	1 eV/molecule	=	96 485 kJ mol <sup>-1</sup>

Prefixes	f	p	n	$\mu$	m	c	d	k	M	G
	femto	pico	nano	micro	milli	centi	deci	kilo	mega	giga
	$10^{-15}$	$10^{-12}$	$10^{-9}$	$10^{-6}$	$10^{-3}$	$10^{-2}$	$10^{-1}$	$10^3$	$10^6$	$10^9$

# PERIODIC TABLE OF ELEMENTS

## GROUPS

PERIODS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	IA	IIA	IIIB	IVB	VB	VIB	VII B	VIII B			IB	II B	IIIA	IVA	VA	VIA	VIIA	VIIIA
1	1.008 H 1																	4.003 He 2
2	6.941 Li 3	9.012 Be 4											Atomic mass → 10.811	12.011	14.007	15.999	18.998	20.180
													Symbol → B	C	N	O	F	Ne
													Atomic No. → 5	6	7	8	9	10
3	22.990 Na 11	24.305 Mg 12	TRANSITION ELEMENTS										26.982 Al 13	28.086 Si 14	30.974 P 15	32.06 S 16	35.453 Cl 17	39.948 Ar 18
4	39.098 K 19	40.078 Ca 20	44.956 Sc 21	47.88 Ti 22	50.942 V 23	51.996 Cr 24	54.938 Mn 25	55.847 Fe 26	58.933 Co 27	58.69 Ni 28	63.546 Cu 29	65.39 Zn 30	69.723 Ga 31	72.61 Ge 32	74.922 As 33	78.96 Se 34	79.904 Br 35	83.80 Kr 36
5	85.468 Rb 37	87.62 Sr 38	88.906 Y 39	91.224 Zr 40	92.906 Nb 41	95.94 Mo 42	98.907 Tc 43	101.07 Ru 44	102.91 Rh 45	106.42 Pd 46	107.87 Ag 47	112.41 Cd 48	114.82 In 49	118.71 Sn 50	121.75 Sb 51	127.60 Te 52	126.90 I 53	131.29 Xe 54
6	132.91 Cs 55	137.33 Ba 56	138.91 *La 57	178.49 Hf 72	180.95 Ta 73	183.85 W 74	186.21 Re 75	190.2 Os 76	192.22 Ir 77	195.08 Pt 78	196.97 Au 79	200.59 Hg 80	204.38 Tl 81	207.2 Pb 82	208.98 Bi 83	(209) Po 84	(210) At 85	(222) Rn 86
7	223 Fr 87	226.03 Ra 88	(227) **Ac 89	(261) Rf 104	(262) Ha 105	(263) Unh 106	(262) Uns 107	(265) Uno 108	(266) Une 109	(267) Uun 110								

\*Lanthanide Series

140.12 Ce 58	140.91 Pr 59	144.24 Nd 60	(145) Pm 61	150.36 Sm 62	151.96 Eu 63	157.25 Gd 64	158.93 Tb 65	162.50 Dy 66	164.93 Ho 67	167.26 Er 68	168.93 Tm 69	173.04 Yb 70	174.97 Lu 71
232.04 Th 90	231.04 Pa 91	238.03 U 92	237.05 Np 93	(244) Pu 94	(243) Am 95	(247) Cm 96	(247) Bk 97	(251) Cf 98	(252) Es 99	(257) Fm 100	(258) Md 101	(259) No 102	(260) Lr 103

\*\*Actinide Series

( ) indicates the mass number of the isotope with the longest half-life.