

**UNIVERSITY OF SWAZILAND
MAIN EXAMINATION PAPER 2010/2011**

TITLE OF PAPER: BIostatISTICS

COURSE CODE: B305

TIME ALLOWED: THREE (3) HOURS

- INSTRUCTIONS:**
1. ANSWER ANY FOUR QUESTIONS.
 2. EACH QUESTION CARRIES TWENTY FIVE (25) MARKS.
 3. ILLUSTRATE YOUR ANSWERS WITH LARGE AND CLEARLY LABELED DIAGRAMS WHERE APPROPRIATE.
 4. CLEARLY STATE YOUR NULL AND ALTERNATIVE HYPOTHESES AND YOUR CONCLUSIONS WHERE APPROPRIATE.

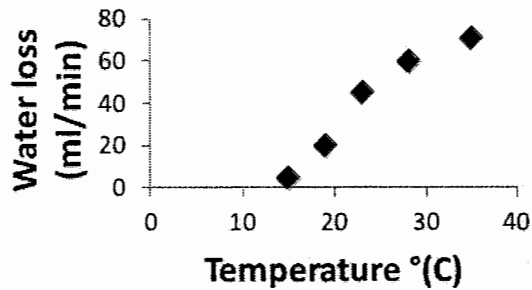
SPECIAL REQUIREMENTS:

1. CALCULATORS (CANDIDATES MUST BRING THEIR OWN).
2. GRAPH PAPER.
3. STATISTICAL TABLES (TO BE SUPPLIED BY THE LECTURER).
4. USEFUL EQUATIONS (TO BE SUPPLIED BY THE LECTURER).

ANSWER ANY FOUR (4) OUT OF SIX (6) QUESTIONS

QUESTION 1

The following data were collected by an ecophysiologist working on water loss in a cold-adapted rodent species:



- a) Will the correlation coefficient r be closer to zero or closer to one (do NOT calculate it!)? Give reasons for your answer. [3 marks]
- b) Would you recommend doing a regression analysis on these data? Clearly explain your decision. [7 marks]
- c) What type of data are presented here? Give reasons for your answer. [4 marks]
- d) Is it possible to predict water loss at 50°C from these data? Elaborate. [5 marks]
- e) What are the assumptions of a parametric test? [6 marks]

[TOTAL = 25 marks]

QUESTION 2

- a) The data given in the table below are normally distributed. Can you use a parametric test to establish whether the two samples come from the same population? Support your answer with appropriate evidence. [4 marks]

	Length (mm)	
	Lowlands	Highlands
	23	24
	27	29
	22	32
	18	31
	24	30
Mean	22.8	29.2
Variance	10.70	9.70

- b) Using an appropriate statistical test, establish whether the lengths of organisms from the lowlands are different from those of the highlands. [10 marks]
- c) Draw a box plot showing the mean, SE and range of these two samples. [11 marks]

[TOTAL = 25 marks]

QUESTION 3

The following table shows the results of vegetation transects in 3 habitats during which grass species were counted. The data do not meet the assumptions of parametric tests.

	Habitat		
	Savanna	Wetland	Grassland
	7	4	10
	8	6	10
	6	3	8
	5	5	7
	7	4	11

- a) Using an appropriate statistical test, establish whether the three different habitats have significantly different grass species richness. [15 marks]
- b) Present these data in the form of a bar graph showing the mean $\pm$ SD [10 marks]

[Total = 25 marks]

QUESTION 4

Consider the following data on male and female school children in a remote part of Swaziland

Sex	Numbers		
	Healthy	Relatively unhealthy	Severely unhealthy
Male	62	23	8
Female	65	26	24

- a) Use an appropriate statistical test to test whether male and female school children differ in their health status. [12 marks]
- b) What are the assumptions of the chi-square test? [3 marks]
- c) Present these data in an appropriate graph that shows the relative abundance of males and females that are healthy, relatively unhealthy and severely unhealthy. [10 marks]

[TOTAL = 25 marks]

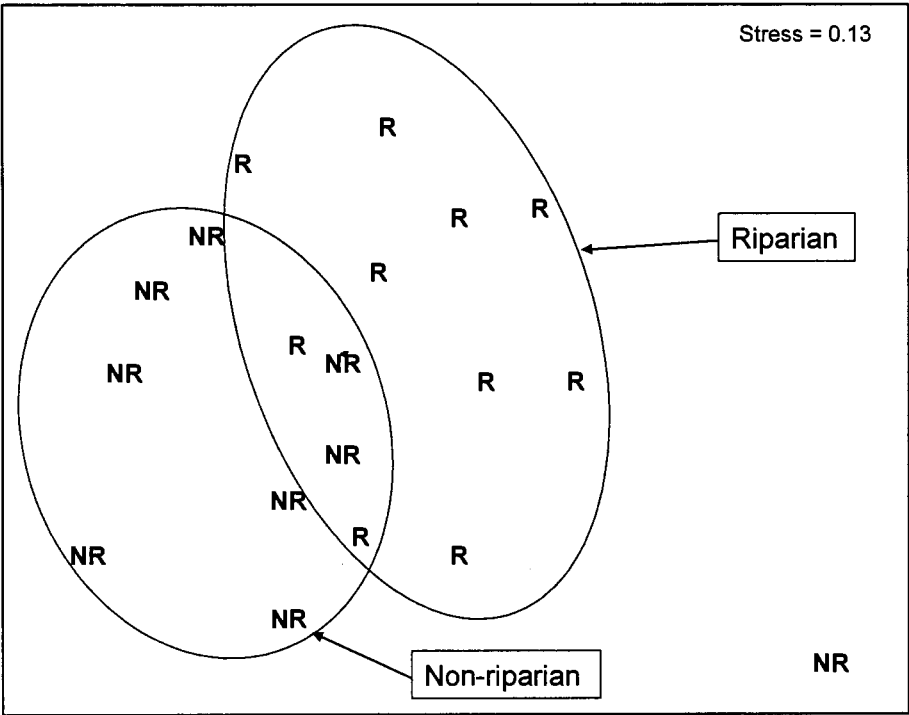
QUESTION 5

A biologist is hired to conduct a survey of a particular species of antelope in a national park in Swaziland. The park has a variety of different habitats, two of which are unsuitable for this antelope: 1) rocky outcrops and 2) open grasslands. Yet these two habitats cover over 75% of the national park. The biologist randomly selects 20 transects across the entire park and uses these to estimate the population of this antelope in the national park.

- a) What is wrong with the design of this survey? [5 marks]
- b) How would you design this survey, if you were given the task of estimating the population of this antelope in the national park? Clearly state your reasons. [10 marks]
- c) What is the difference between accuracy and precision? [5 marks]
- d) How could you increase precision in the example above? Elaborate. [5 marks]

QUESTION 6

- a) Define multivariate statistics? [3 marks]
- b) How does Principal Components Analysis (PCA) differ from Cluster Analysis?
Explain the outputs of each analysis (you do not need to elaborate on how the analyses are performed), and describe the situations when you would use PCA and when you would use Cluster Analysis. [12 marks]
- c) The figure below was produced from a non-metric Multi-Dimensional Scaling (MDS) analysis. The analysis was based on the survey of butterfly species at 19 different sites in two habitats: 10 sites in riparian (R) and 9 sites in non-riparian habitats (NR). What conclusions can you draw from this figure? [10 marks]



B305 (BIOSTATISTICS) EXAMINATION

USEFUL EQUATIONS

$$S^2 = \sum (y_i - \bar{y})^2 / (n-1)$$

$$SE = S / \sqrt{n}$$

$$Z = (y_i - \mu) / \sigma$$

$$Z = (\bar{y} - \mu) / SE$$

$$C.I.: \bar{y} \pm [t_{(\alpha, df)}] SE$$

$$t = (\bar{y} - \mu) / SE$$

$$t = (\bar{y}_1 - \bar{y}_2) / SE_{pooled}$$

$$SE_{pooled} = SD_{pooled} \sqrt{(1/n_1 + 1/n_2)}$$

$$SD_{pooled} = \sqrt{[(df_1 s_1^2 + df_2 s_2^2) / (df_1 + df_2)]}$$

$$\chi^2 = \sum (O - E)^2 / E$$

$$\chi^2 = [n(|cell1 * cell4 - cell2 * cell3| - n/2)^2] / [C_1 * C_2 * R_1 * R_2]$$

$$\text{Total SS} = \sum \sum y^2 - (\sum \sum y)^2 / N$$

$$\text{Groups SS} = \sum (\sum y)^2 / n - (\sum \sum y)^2 / N$$

$$\text{Error SS} = \text{Total SS} - \text{Groups SS}$$

$$F = \text{Groups MS} / \text{Error MS}$$

$$q = (\bar{y}_a - \bar{y}_b) / SE$$

$$y' = \log(y+1)$$

$$y' = \sqrt{(y+0.5)}$$

$$y' = \arcsin \sqrt{y}$$

$$\Phi = \sqrt{[(k-1)(\text{groups MS} - \text{error MS})] / (k)(\text{error MS})}$$

$$U = n_1 n_2 + n_1(n_1+1)/2 - R_1$$

$$H = 12/N(N+1) \sum [R^2/n] - 3(N+1)$$

$$r = \sum xy / \sqrt{(\sum x^2)(\sum y^2)}$$

$$b = \sum xy / \sum x^2$$

$$\sum xy = \sum XY - (\sum X)(\sum Y)/n$$

$$\sum x^2 = \sum X^2 - (\sum X)^2/n$$

$$\sum y^2 = \sum Y^2 - (\sum Y)^2/n$$

$$\text{Total SS} = \sum y^2$$

$$\text{Regression SS} = (\sum xy)^2 / \sum x^2$$

$$\text{Residual SS} = \text{Total SS} - \text{Regression SS}$$

$$P(X) = n! / X!(n-X)! [p^x (q^{n-x})]$$

$$p = [\sum (X)(\text{Obs}) / \sum (\text{Obs})] / n$$

$$P(X) = \mu^x / e^\mu (X!)$$

$$\mu = [\sum (X * \text{Obs})] / [\sum \text{Obs}]$$

TABLE B.3 CRITICAL VALUES OF THE t DISTRIBUTION

ν	$\alpha(2):$ 0.50	0.20	0.10	0.05	0.02	0.01	0.005	0.002	
	$\alpha(1):$ 0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	
1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	63
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	3
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	1
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.393	
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	
31	0.682	1.309	1.696	2.040	2.453	2.744	3.022	3.375	
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	
33	0.682	1.308	1.692	2.035	2.445	2.733	3.008	3.356	
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	
35	0.682	1.306	1.690	2.030	2.438	2.724	2.996	3.340	
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	
37	0.681	1.305	1.687	2.026	2.431	2.715	2.985	3.326	
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	
39	0.681	1.304	1.685	2.023	2.426	2.708	2.976	3.313	
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	
41	0.681	1.303	1.683	2.020	2.421	2.701	2.967	3.301	
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	
43	0.680	1.302	1.681	2.017	2.416	2.695	2.959	3.291	
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	
45	0.680	1.301	1.679	2.014	2.412	2.690	2.952	3.281	
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	
47	0.680	1.300	1.678	2.012	2.408	2.685	2.946	3.273	
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	
49	0.680	1.299	1.677	2.010	2.405	2.680	2.940	3.265	
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	

TABLE B.16 CRITICAL VALUES OF THE CORRELATION COEFFICIENT, r

$\alpha(2):$	0.50	0.20	0.10	0.05	0.02	0.01	0.005	0.002	0.001
$\alpha(1):$	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
v									
1	0.707	0.951	0.988	0.997	1.000	1.000	1.000	1.000	1.000
2	0.500	0.800	0.900	0.950	0.980	0.990	0.995	0.998	0.999
3	0.404	0.687	0.805	0.878	0.934	0.959	0.974	0.986	0.991
4	0.347	0.608	0.729	0.811	0.882	0.917	0.942	0.963	0.974
5	0.309	0.551	0.669	0.755	0.833	0.875	0.906	0.935	0.951
6	0.281	0.507	0.621	0.707	0.789	0.834	0.870	0.905	0.925
7	0.260	0.472	0.582	0.666	0.750	0.798	0.836	0.875	0.898
8	0.242	0.443	0.549	0.632	0.715	0.765	0.805	0.847	0.872
9	0.228	0.419	0.521	0.602	0.685	0.735	0.776	0.820	0.847
10	0.216	0.398	0.497	0.576	0.658	0.708	0.750	0.795	0.823
11	0.206	0.380	0.476	0.553	0.634	0.684	0.726	0.772	0.801
12	0.197	0.365	0.457	0.532	0.612	0.661	0.703	0.750	0.780
13	0.189	0.351	0.441	0.514	0.592	0.641	0.683	0.730	0.760
14	0.182	0.338	0.426	0.497	0.574	0.623	0.664	0.711	0.742
15	0.176	0.327	0.412	0.482	0.558	0.606	0.647	0.694	0.725
16	0.170	0.317	0.400	0.468	0.542	0.590	0.631	0.678	0.708
17	0.165	0.308	0.389	0.456	0.529	0.575	0.616	0.662	0.693
18	0.160	0.299	0.378	0.444	0.515	0.561	0.602	0.648	0.679
19	0.156	0.291	0.369	0.433	0.503	0.549	0.589	0.635	0.666
20	0.152	0.284	0.360	0.423	0.492	0.537	0.576	0.622	0.652
21	0.148	0.277	0.352	0.413	0.482	0.526	0.565	0.610	0.640
22	0.145	0.271	0.344	0.404	0.472	0.515	0.554	0.599	0.629
23	0.141	0.265	0.337	0.396	0.462	0.505	0.543	0.588	0.618
24	0.138	0.260	0.330	0.388	0.453	0.496	0.534	0.578	0.607
25	0.136	0.255	0.323	0.381	0.445	0.487	0.524	0.568	0.597
26	0.133	0.250	0.317	0.374	0.437	0.479	0.515	0.559	0.588
27	0.131	0.245	0.311	0.367	0.430	0.471	0.507	0.550	0.579
28	0.128	0.241	0.306	0.361	0.423	0.463	0.499	0.541	0.570
29	0.126	0.237	0.301	0.355	0.416	0.456	0.491	0.533	0.562
30	0.124	0.233	0.296	0.349	0.409	0.449	0.484	0.526	0.554
31	0.122	0.229	0.291	0.344	0.403	0.442	0.477	0.518	0.546
32	0.120	0.225	0.287	0.339	0.397	0.436	0.470	0.511	0.539
33	0.118	0.222	0.283	0.334	0.392	0.430	0.464	0.504	0.532
34	0.116	0.219	0.279	0.329	0.386	0.424	0.458	0.498	0.525
35	0.115	0.216	0.275	0.325	0.381	0.418	0.452	0.492	0.519
36	0.113	0.213	0.271	0.320	0.376	0.413	0.446	0.486	0.513
37	0.111	0.210	0.267	0.316	0.371	0.408	0.441	0.480	0.507
38	0.110	0.207	0.264	0.312	0.367	0.403	0.435	0.474	0.501
39	0.108	0.204	0.261	0.308	0.362	0.398	0.430	0.469	0.495
40	0.107	0.202	0.257	0.304	0.358	0.393	0.425	0.463	0.490
41	0.106	0.199	0.254	0.301	0.354	0.389	0.420	0.458	0.484
42	0.104	0.197	0.251	0.297	0.350	0.384	0.416	0.453	0.479
43	0.103	0.195	0.248	0.294	0.346	0.380	0.411	0.449	0.474
44	0.102	0.192	0.246	0.291	0.342	0.376	0.407	0.444	0.469
45	0.101	0.190	0.243	0.288	0.339	0.372	0.403	0.439	0.465
46	0.100	0.188	0.240	0.285	0.335	0.368	0.399	0.435	0.460
47	0.099	0.186	0.238	0.282	0.331	0.365	0.395	0.431	0.456
48	0.098	0.184	0.235	0.279	0.328	0.361	0.391	0.427	0.451
49	0.097	0.182	0.233	0.276	0.325	0.358	0.387	0.423	0.447
50	0.096	0.181	0.231	0.273	0.322	0.354	0.384	0.419	0.443

TABLE B.5 (cont.) CRITICAL VALUES OF THE q DISTRIBUTION
 $\alpha = 0.05$

v	$k(\text{or } p) = 2$	3	4	5	6	7	8	9	10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.085	8.331	9.798	10.88	11.74	12.44	13.03	13.54	13.99
3	4.501	5.910	6.825	7.502	8.037	8.478	8.853	9.177	9.462
4	3.927	5.040	5.757	6.287	6.707	7.053	7.347	7.602	7.826
5	3.635	4.602	5.218	5.673	6.033	6.330	6.582	6.802	6.995
6	3.461	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493
7	3.344	4.165	4.681	5.060	5.359	5.606	5.815	5.998	6.158
8	3.261	4.041	4.529	4.886	5.167	5.399	5.597	5.767	5.918
9	3.199	3.949	4.415	4.756	5.024	5.244	5.432	5.595	5.739
10	3.151	3.877	4.327	4.654	4.912	5.124	5.305	5.461	5.599
11	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.487
12	3.082	3.773	4.199	4.508	4.751	4.950	5.119	5.265	5.395
13	3.055	3.735	4.151	4.453	4.690	4.885	5.049	5.192	5.318
14	3.033	3.702	4.111	4.407	4.639	4.829	4.990	5.131	5.254
15	3.014	3.674	4.076	4.367	4.595	4.782	4.940	5.077	5.198
16	2.998	3.649	4.046	4.333	4.557	4.741	4.897	5.031	5.150
17	2.984	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108
18	2.971	3.609	3.997	4.277	4.495	4.673	4.824	4.956	5.071
19	2.960	3.593	3.977	4.253	4.469	4.645	4.794	4.924	5.038
20	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.896	5.008
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
30	2.888	3.486	3.845	4.102	4.302	4.464	4.602	4.720	4.824
40	2.858	3.442	3.791	4.039	4.232	4.389	4.521	4.635	4.735
60	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646
120	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560
∞	2.772	3.314	3.633	3.858	4.030	4.170	4.286	4.387	4.474

v	$k(\text{or } p) = 11$	12	13	14	15	16	17	18	19
1	50.59	51.96	53.20	54.33	55.36	56.32	57.22	58.04	58.83
2	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.37	16.57
3	9.717	9.946	10.15	10.35	10.53	10.69	10.84	10.98	11.11
4	8.027	8.208	8.373	8.525	8.664	8.794	8.914	9.028	9.134
5	7.168	7.324	7.466	7.596	7.717	7.828	7.932	8.030	8.122
6	6.649	6.789	6.917	7.034	7.143	7.244	7.338	7.426	7.508
7	6.302	6.431	6.550	6.658	6.759	6.852	6.939	7.020	7.097
8	6.054	6.175	6.287	6.389	6.483	6.571	6.653	6.729	6.802
9	5.867	5.983	6.089	6.186	6.276	6.359	6.437	6.510	6.579
10	5.722	5.833	5.935	6.028	6.114	6.194	6.269	6.339	6.405
11	5.605	5.713	5.811	5.901	5.984	6.062	6.134	6.202	6.265
12	5.511	5.615	5.710	5.798	5.878	5.953	6.023	6.089	6.151
13	5.431	5.533	5.625	5.711	5.789	5.862	5.931	5.995	6.055
14	5.364	5.463	5.554	5.637	5.714	5.786	5.852	5.915	5.974
15	5.306	5.404	5.493	5.574	5.649	5.720	5.785	5.846	5.904
16	5.256	5.352	5.439	5.520	5.593	5.662	5.727	5.786	5.843
17	5.212	5.307	5.392	5.471	5.544	5.612	5.675	5.734	5.790
18	5.174	5.267	5.352	5.429	5.501	5.568	5.630	5.688	5.743
19	5.140	5.231	5.315	5.391	5.462	5.528	5.589	5.647	5.701
20	5.108	5.199	5.282	5.357	5.427	5.493	5.553	5.610	5.663
24	5.012	5.099	5.179	5.251	5.319	5.381	5.439	5.494	5.545
30	4.917	5.001	5.077	5.147	5.211	5.271	5.327	5.379	5.429
40	4.824	4.904	4.977	5.044	5.106	5.163	5.216	5.266	5.313
60	4.732	4.808	4.878	4.942	5.001	5.056	5.107	5.154	5.199
120	4.641	4.714	4.781	4.842	4.898	4.950	4.998	5.044	5.086
∞	4.552	4.622	4.685	4.743	4.796	4.845	4.891	4.934	4.974

TABLE B.4 CRITICAL VALUES OF THE F DISTRIBUTION

		Numerator DF = 1									
$\alpha(2):$		0.50	0.20	0.10	0.05	0.02	0.01	0.005	0.002	0.001	
$\alpha(1):$		0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005	
Denom. DF											
1		5.83	39.9	161.	648.	4050.	16200.	64800.	405000.	1620000.	
2		2.57	8.53	18.5	38.5	98.5	199.	399.	999.	2000.	
3		2.02	5.54	10.1	17.4	34.1	55.6	89.6	167.	267.	
4		1.81	4.54	7.71	12.2	21.2	31.3	45.7	74.1	106.	
5		1.69	4.06	6.61	10.0	16.3	22.8	31.4	47.2	63.6	
6		1.62	3.78	5.99	8.81	13.7	18.6	24.8	35.5	46.1	
7		1.57	3.59	5.59	8.07	12.2	16.2	21.1	29.2	37.0	
8		1.54	3.46	5.32	7.57	11.3	14.7	18.8	25.4	31.6	
9		1.51	3.36	5.12	7.21	10.6	13.6	17.2	22.9	28.0	
10		1.49	3.29	4.96	6.94	10.0	12.8	16.0	21.0	25.5	
11		1.47	3.23	4.84	6.72	9.65	12.2	15.2	19.7	23.7	
12		1.46	3.18	4.75	6.55	9.33	11.8	14.5	18.6	22.2	
13		1.45	3.14	4.67	6.41	9.07	11.4	13.9	17.8	21.1	
14		1.44	3.10	4.60	6.30	8.86	11.1	13.5	17.1	20.2	
15		1.43	3.07	4.54	6.20	8.68	10.8	13.1	16.6	19.5	
16		1.42	3.05	4.49	6.12	8.53	10.6	12.8	16.1	18.9	
17		1.42	3.03	4.45	6.04	8.40	10.4	12.6	15.7	18.4	
18		1.41	3.01	4.41	5.98	8.29	10.2	12.3	15.4	17.9	
19		1.41	2.99	4.38	5.92	8.18	10.1	12.1	15.1	17.5	
20		1.40	2.97	4.35	5.87	8.10	9.94	11.9	14.8	17.2	
21		1.40	2.96	4.32	5.83	8.02	9.83	11.8	14.6	16.9	
22		1.40	2.95	4.30	5.79	7.95	9.73	11.6	14.4	16.6	
23		1.39	2.94	4.28	5.75	7.88	9.63	11.5	14.2	16.4	
24		1.39	2.93	4.26	5.72	7.82	9.55	11.4	14.0	16.2	
25		1.39	2.92	4.24	5.69	7.77	9.48	11.3	13.9	16.0	
26		1.38	2.91	4.23	5.66	7.72	9.41	11.2	13.7	15.8	
27		1.38	2.90	4.21	5.63	7.68	9.34	11.1	13.6	15.6	
28		1.38	2.89	4.20	5.61	7.64	9.28	11.0	13.5	15.5	
29		1.38	2.89	4.18	5.59	7.60	9.23	11.0	13.4	15.3	
30		1.38	2.88	4.17	5.57	7.56	9.18	10.9	13.3	15.2	
35		1.37	2.85	4.12	5.48	7.42	8.98	10.6	12.9	14.7	
40		1.36	2.84	4.08	5.42	7.31	8.83	10.4	12.6	14.4	
45		1.36	2.82	4.06	5.38	7.23	8.71	10.3	12.4	14.1	
50		1.35	2.81	4.03	5.34	7.17	8.63	10.1	12.2	13.9	
60		1.35	2.79	4.00	5.29	7.08	8.49	9.96	12.0	13.5	
70		1.35	2.78	3.98	5.25	7.01	8.40	9.84	11.8	13.3	
80		1.34	2.77	3.96	5.22	6.96	8.33	9.75	11.7	13.2	
90		1.34	2.76	3.95	5.20	6.93	8.28	9.68	11.6	13.0	
100		1.34	2.76	3.94	5.18	6.90	8.24	9.62	11.5	12.9	
120		1.34	2.75	3.92	5.15	6.85	8.18	9.54	11.4	12.8	
140		1.33	2.74	3.91	5.13	6.82	8.14	9.48	11.3	12.7	
160		1.33	2.74	3.90	5.12	6.80	8.10	9.44	11.2	12.6	
180		1.33	2.73	3.89	5.11	6.78	8.08	9.40	11.2	12.6	
200		1.33	2.73	3.89	5.10	6.76	8.06	9.38	11.2	12.5	
300		1.33	2.72	3.87	5.07	6.72	8.00	9.30	11.0	12.4	
500		1.33	2.72	3.86	5.05	6.69	7.95	9.23	11.0	12.3	
∞		1.32	2.71	3.84	5.02	6.64	7.88	9.14	10.8	12.1	

TABLE B.4 (cont.) CRITICAL VALUES OF THE F DISTRIBUTION

Station	0.15	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35	1.50	1.65	1.80	1.95	2.10	2.25	2.40	2.55	2.70	2.85	3.00	3.15	3.30	3.45	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65	4.80	4.95	5.10	5.25	5.40	5.55	5.70	5.85	6.00	6.15	6.30	6.45	6.60	6.75	6.90	7.05	7.20	7.35	7.50	7.65	7.80	7.95	8.10	8.25	8.40	8.55	8.70	8.85	9.00	9.15	9.30	9.45	9.60	9.75	9.90	10.05	10.20	10.35	10.50	10.65	10.80	10.95	11.10	11.25	11.40	11.55	11.70	11.85	12.00	12.15	12.30	12.45	12.60	12.75	12.90	13.05	13.20	13.35	13.50	13.65	13.80	13.95	14.10	14.25	14.40	14.55	14.70	14.85	15.00	15.15	15.30	15.45	15.60	15.75	15.90	16.05	16.20	16.35	16.50	16.65	16.80	16.95	17.10	17.25	17.40	17.55	17.70	17.85	18.00	18.15	18.30	18.45	18.60	18.75	18.90	19.05	19.20	19.35	19.50	19.65	19.80	19.95	20.10	20.25	20.40	20.55	20.70	20.85	21.00	21.15	21.30	21.45	21.60	21.75	21.90	22.05	22.20	22.35	22.50	22.65	22.80	22.95	23.10	23.25	23.40	23.55	23.70	23.85	24.00	24.15	24.30	24.45	24.60	24.75	24.90	25.05	25.20	25.35	25.50	25.65	25.80	25.95	26.10	26.25	26.40	26.55	26.70	26.85	27.00	27.15	27.30	27.45	27.60	27.75	27.90	28.05	28.20	28.35	28.50	28.65	28.80	28.95	29.10	29.25	29.40	29.55	29.70	29.85	30.00	30.15	30.30	30.45	30.60	30.75	30.90	31.05	31.20	31.35	31.50	31.65	31.80	31.95	32.10	32.25	32.40	32.55	32.70	32.85	33.00	33.15	33.30	33.45	33.60	33.75	33.90	34.05	34.20	34.35	34.50	34.65	34.80	34.95	35.10	35.25	35.40	35.55	35.70	35.85	36.00	36.15	36.30	36.45	36.60	36.75	36.90	37.05	37.20	37.35	37.50	37.65	37.80	37.95	38.10	38.25	38.40	38.55	38.70	38.85	39.00	39.15	39.30	39.45	39.60	39.75	39.90	40.05	40.20	40.35	40.50	40.65	40.80	40.95	41.10	41.25	41.40	41.55	41.70	41.85	42.00	42.15	42.30	42.45	42.60	42.75	42.90	43.05	43.20	43.35	43.50	43.65	43.80	43.95	44.10	44.25	44.40	44.55	44.70	44.85	45.00	45.15	45.30	45.45	45.60	45.75	45.90	46.05	46.20	46.35	46.50	46.65	46.80	46.95	47.10	47.25	47.40	47.55	47.70	47.85	48.00	48.15	48.30	48.45	48.60	48.75	48.90	49.05	49.20	49.35	49.50	49.65	49.80	49.95	50.10	50.25	50.40	50.55	50.70	50.85	51.00	51.15	51.30	51.45	51.60	51.75	51.90	52.05	52.20	52.35	52.50	52.65	52.80	52.95	53.10	53.25	53.40	53.55	53.70	53.85	54.00	54.15	54.30	54.45	54.60	54.75	54.90	55.05	55.20	55.35	55.50	55.65	55.80	55.95	56.10	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.45	57.60	57.75	57.90	58.05	58.20	58.35	58.50	58.65	58.80	58.95	59.10	59.25	59.40	59.55	59.70	59.85	60.00	60.15	60.30	60.45	60.60	60.75	60.90	61.05	61.20	61.35	61.50	61.65	61.80	61.95	62.10	62.25	62.40	62.55	62.70	62.85	63.00	63.15	63.30	63.45	63.60	63.75	63.90	64.05	64.20	64.35	64.50	64.65	64.80	64.95	65.10	65.25	65.40	65.55	65.70	65.85	66.00	66.15	66.30	66.45	66.60	66.75	66.90	67.05	67.20	67.35	67.50	67.65	67.80	67.95	68.10	68.25	68.40	68.55	68.70	68.85	69.00	69.15	69.30	69.45	69.60	69.75	69.90	70.05	70.20	70.35	70.50	70.65	70.80	70.95	71.10	71.25	71.40	71.55	71.70	71.85	72.00	72.15	72.30	72.45	72.60	72.75	72.90	73.05	73.20	73.35	73.50	73.65	73.80	73.95	74.10	74.25	74.40	74.55	74.70	74.85	75.00	75.15	75.30	75.45	75.60	75.75	75.90	76.05	76.20	76.35	76.50	76.65	76.80	76.95	77.10	77.25	77.40	77.55	77.70	77.85	78.00	78.15	78.30	78.45	78.60	78.75	78.90	79.05	79.20	79.35	79.50	79.65	79.80	79.95	80.10	80.25	80.40	80.55	80.70	80.85	81.00	81.15	81.30	81.45	81.60	81.75	81.90	82.05	82.20	82.35	82.50	82.65	82.80	82.95	83.10	83.25	83.40	83.55	83.70	83.85	84.00	84.15	84.30	84.45	84.60	84.75	84.90	85.05	85.20	85.35	85.50	85.65	85.80	85.95	86.10	86.25	86.40	86.55	86.70	86.85	87.00	87.15	87.30	87.45	87.60	87.75	87.90	88.05	88.20	88.35	88.50	88.65	88.80	88.95	89.10	89.25	89.40	89.55	89.70	89.85	90.00	90.15	90.30	90.45	90.60	90.75	90.90	91.05	91.20	91.35	91.50	91.65	91.80	91.95	92.10	92.25	92.40	92.55	92.70	92.85	93.00	93.15	93.30	93.45	93.60	93.75	93.90	94.05	94.20	94.35	94.50	94.65	94.80	94.95	95.10	95.25	95.40	95.55	95.70	95.85	96.00	96.15	96.30	96.45	96.60	96.75	96.90	97.05	97.20	97.35	97.50	97.65	97.80	97.95	98.10	98.25	98.40	98.55	98.70	98.85	99.00	99.15	99.30	99.45	99.60	99.75	99.90	100.05	100.20	100.35	100.50	100.65	100.80	100.95	101.10	101.25	101.40	101.55	101.70	101.85	102.00	102.15	102.30	102.45	102.60	102.75	102.90	103.05	103.20	103.35	103.50	103.65	103.80	103.95	104.10	104.25	104.40	104.55	104.70	104.85	105.00	105.15	105.30	105.45	105.60	105.75	105.90	106.05	106.20	106.35	106.50	106.65	106.80	106.95	107.10	107.25	107.40	107.55	107.70	107.85	108.00	108.15	108.30	108.45	108.60	108.75	108.90	109.05	109.20	109.35	109.50	109.65	109.80	109.95	110.10	110.25	110.40	110.55	110.70	110.85	111.00	111.15	111.30	111.45	111.60	111.75	111.90	112.05	112.20	112.35	112.50	112.65	112.80	112.95	113.10	113.25	113.40	113.55	113.70	113.85	114.00	114.15	114.30	114.45	114.60	114.75	114.90	115.05	115.20	115.35	115.50	115.65	115.80	115.95	116.10	116.25	116.40	116.55	116.70	116.85	117.00	117.15	117.30	117.45	117.60	117.75	117.90	118.05	118.20	118.35	118.50	118.65	118.80	118.95	119.10	119.25	119.40	119.55	119.70	119.85	120.00	120.15	120.30	120.45	120.60	120.75	120.90	121.05	121.20	121.35	121.50	121.65	121.80	121.95	122.10	122.25	122.40	122.55	122.70	122.85	123.00	123.15	123.30	123.45	123.60	123.75	123.90	124.05	124.20	124.35	124.50	124.65	124.80	124.95	125.10	125.25	125.40	125.55	125.70	125.85	126.00	126.15	126.30	126.45	126.60	126.75	126.90	127.05	127.20	127.35	127.50	127.65	127.80	127.95	128.10	128.25	128.40	128.55	128.70	128.85	129.00	129.15	129.30	129.45	129.60	129.75	129.90	130.05	130.20	130.35	130.50	130.65	130.80	130.95	131.10	131.25	131.40	131.55	131.70	131.85	132.00	132.15	132.30	132.45	132.60	132.75	132.90	133.05	133.20	133.35	133.50	133.65	133.80	133.95	134.10	134.25	134.40	134.55	134.70	134.85	135.00	135.15	135.30	135.45	135.60	135.75	135.90	136.05	136.20	136.35	136.50	136.65	136.80	136.95	137.10	137.25	137.40	137.55	137.70	137.85	138.00	138.15	138.30	138.45	138.60	138.75	138.90	139.05	139.20	139.35	139.50	139.65	139.80	139.95	140.10	140.25	140.40	140.55	140.70	140.85	141.00	141.15	141.30	141.45	141.60	141.75	141.90	142.05	142.20	142.35	142.50	142.65	142.80	142.95	143.10	143.25	143.40	143.55	143.70	143.85	144.00	144.15	144.30	144.45	144.60	144.75	144.90	145.05	145.20	145.35	145.50	145.65	145.80	145.95	146.10	146.25	146.40	146.55	146.70	146.85	147.00	147.15	147.30	147.45	147.60	147.75	147.90	148.05	148.20	148.35	148.50	148.65	148.80	148.95	149.10	149.25	149.40	149.55	149.70	149.85	150.00	150.15	150.30	150.45	150.60	150.75	150.90	151.05	151.20	151.35	151.50	151.65	151.80	151.95	152.10	152.25	152.40	152.55	152.70	152.85	153.00	153.15	153.30	153.45	153.60	153.75	153.90	154.05	154.20	154.35	154.50	154.65	154.80	154.95	155.10	155.25	155.40	155.55	155.70	155.85	156.00	156.15	156.30	156.45	156.60	156.75	156.90	157.05	157.20	157.35	157.50	157.65	157.80	157.95	158.10	158.25	158.40	158.55	158.70	158.85	159.00	159.15	159.30	159.45	159.60	159.75	159.90	160.05	160.20	160.35	160.50	160.65	160.80	160.95	161.10	161.25	161.40	161.55	161.70	161.85	162.00	162.15	162.30	162.45	162.60	162.75	162.90	163.05	163.20	163.35	163.50	163.65	163.80	163.95	164.10	164.25	164.40	164.55	164.70	164.85	165.00	165.15	165.30	165.45	165.60	165.75	165.90	166.05	166.20	166.35	166.50	166.65	166.80	166.95	167.10	167.25	167.40	167.55	167.70	167.85	168.00	168.15	168.30	168.45	168.60	168.75	168.90	169.05	169.20	169.35	169.50	169.65	169.80	169.95	170.10	170.25	170.40	170.55	170.70	170.85	171.00	171.15	171.30	171.45	171.60	171.75	171.90	172.05	172.20	172.35	172.50	172.65	172.80	172.95	173.10	173.25	173.40	173.55	173.70	173.85	174.00	174.15	174.30	174.45	174.60	174.75	174.90	175.05	175.20	175.35	175.50	175.65	175.80	175.95	176.10	176.25	176.40	176.55	176.70	176.85	177.00	177.15	177.30	177.45	177.60	177.75	177.90	178.05	178.20	178.35	178.50	178.65	178
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TABLE B.4 (cont.) CRITICAL VALUES OF THE F DISTRIBUTION

[illegible]

TABLE B.4 (cont.) CRITICAL VALUES OF THE F DISTRIBUTION

		Number of π									
π^0	$\pi^\pm$	0	1	2	3	4	5	6	7	8	9
1	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
2	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
3	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
4	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
5	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
6	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
7	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
8	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
9	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
10	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
11	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
12	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
13	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
14	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
15	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
16	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
17	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
18	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
19	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
20	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
21	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
22	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
23	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
24	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
25	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
26	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
27	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
28	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
29	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
31	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00
32	0.11	0.26	0.20	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 1. CRITICAL VALUES OF THE CHI-SQUARE DISTRIBUTION

ν	$\alpha = 0.999$	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005	0.001
1	0.000	0.000	0.000	0.001	0.004	0.016	0.102	0.455	1.323	2.706	3.841	5.024	6.635	7.879	10.828
2	0.002	0.010	0.020	0.051	0.103	0.211	0.575	1.386	2.773	4.605	5.991	7.378	9.210	10.597	13.816
3	0.024	0.072	0.150	0.352	0.484	0.711	1.213	2.366	4.108	6.251	7.815	9.348	11.345	12.838	16.266
4	0.091	0.207	0.287	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	11.143	13.277	14.860	18.467
5	0.210	0.412	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.070	12.833	15.086	16.750	20.515
6	0.381	0.676	0.872	1.237	1.635	2.204	3.455	5.348	7.884	10.645	12.592	14.449	16.812	18.548	22.458
7	0.559	0.989	1.353	1.690	2.167	2.833	4.235	6.346	9.037	12.017	14.067	16.013	18.475	20.278	24.352
8	0.857	1.344	1.846	2.180	2.733	3.591	5.071	7.344	10.219	13.361	15.507	17.535	20.090	21.955	26.124
9	1.152	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.389	14.684	16.919	19.023	21.666	23.589	27.877
10	1.479	2.156	2.558	3.247	3.940	4.865	6.757	9.342	12.549	15.987	18.307	20.483	23.209	25.188	29.588
11	1.834	2.603	3.053	3.816	4.575	5.578	7.584	10.541	13.701	17.275	19.675	21.920	24.725	26.757	31.264
12	2.214	3.074	3.571	4.409	5.226	6.314	8.438	11.440	14.845	18.582	21.026	23.337	26.217	28.300	32.909
13	2.617	3.565	4.107	5.004	5.892	7.042	9.299	12.362	15.984	19.812	22.362	24.736	27.688	29.819	34.528
14	3.041	4.075	4.660	5.629	6.571	7.780	10.165	13.339	17.117	21.064	23.685	26.119	29.141	31.319	36.123
15	3.483	4.601	5.229	6.262	7.261	8.547	11.037	14.339	18.245	22.307	24.996	27.488	30.578	32.801	37.697
16	3.942	5.142	5.812	6.908	7.962	9.312	11.912	15.338	19.369	23.542	26.296	28.845	32.000	34.267	39.252
17	4.416	5.697	6.408	7.564	8.672	10.085	12.792	16.338	20.489	24.769	27.587	30.191	33.409	35.718	40.790
18	4.905	6.265	7.015	8.231	9.590	10.885	13.675	17.338	21.605	25.989	28.869	31.526	34.805	37.156	42.312
19	5.407	6.844	7.633	8.907	10.117	11.651	14.562	18.338	22.718	27.204	30.144	32.852	36.191	38.582	43.820
20	5.921	7.434	8.260	9.591	10.851	12.443	15.452	19.337	23.828	28.412	31.410	34.170	37.566	39.997	45.315
21	6.447	8.034	8.897	10.283	11.591	13.240	16.344	20.337	24.935	29.615	32.671	35.479	38.932	41.401	46.797
22	6.983	8.643	9.542	10.982	12.338	14.041	17.240	21.337	26.039	30.813	33.924	36.781	40.289	42.796	48.268
23	7.529	9.260	10.296	11.689	13.091	14.848	18.137	22.337	27.141	32.007	35.172	38.076	41.538	44.181	49.728
24	8.085	9.886	10.859	12.401	13.848	15.659	19.037	23.337	28.241	33.196	36.415	39.364	42.580	45.559	51.179
25	8.649	10.520	11.524	13.120	14.611	16.473	19.939	24.337	29.339	34.382	37.652	40.646	44.314	46.928	52.620
26	9.222	11.160	12.198	13.844	15.379	17.292	20.843	25.336	30.435	35.563	38.885	41.923	45.642	48.290	54.052
27	9.803	11.808	12.878	14.573	16.151	18.114	21.749	26.336	31.528	36.741	40.113	43.195	46.963	49.645	55.476
28	10.391	12.461	13.565	15.308	16.928	18.938	22.657	27.336	32.620	37.916	41.337	44.461	48.278	50.993	56.892
29	10.986	13.121	14.256	16.047	17.708	19.768	23.567	28.336	33.711	39.087	42.537	45.722	49.588	52.336	58.301
30	11.588	13.787	14.953	16.791	18.493	20.599	24.478	29.336	34.800	40.256	43.773	46.979	50.892	53.672	59.703
31	12.196	14.458	15.655	17.539	19.281	21.434	25.390	30.336	35.887	41.422	44.985	48.232	52.191	55.003	61.098
32	12.811	15.134	16.362	18.291	20.072	22.271	26.304	31.336	36.973	42.585	46.194	49.480	53.486	56.328	62.487
33	13.431	15.815	17.074	19.047	20.867	23.110	27.219	32.336	38.058	43.745	47.400	50.725	54.776	57.648	63.870
34	14.057	16.501	17.789	19.806	21.664	23.952	28.136	33.336	39.141	44.903	48.602	51.966	56.061	58.964	65.247
35	14.688	17.192	18.509	20.569	22.465	24.797	29.054	34.336	40.223	46.059	49.802	53.203	57.342	60.275	66.619
36	15.324	17.887	19.233	21.336	23.269	25.643	29.973	35.336	41.304	47.212	50.998	54.437	58.619	61.581	67.985
37	15.965	18.586	19.960	22.106	24.075	26.492	30.893	36.336	42.383	48.363	52.192	55.668	59.833	62.883	69.346
38	16.611	19.289	20.691	22.878	24.884	27.343	31.815	37.335	43.462	49.513	53.384	56.896	61.162	64.181	70.703
39	17.262	19.996	21.426	23.654	25.695	28.186	32.737	38.335	44.539	50.660	54.572	58.120	62.428	65.476	72.055
40	17.916	20.707	22.164	24.433	26.509	29.051	33.660	39.335	45.616	51.805	55.758	59.342	63.691	66.766	73.402
41	18.576	21.421	22.906	25.215	27.326	29.907	34.585	40.335	46.692	52.949	56.942	60.561	64.950	68.053	74.745
42	19.239	22.138	23.650	25.999	28.144	30.765	35.510	41.335	47.766	54.090	58.124	61.777	66.206	69.336	76.084
43	19.906	22.859	24.398	26.785	28.965	31.625	36.436	42.335	48.840	55.230	59.304	62.990	67.459	70.616	77.419
44	20.576	23.584	25.148	27.575	29.787	32.487	37.363	43.335	49.913	56.369	60.481	64.201	68.710	71.893	78.750
45	21.251	24.311	25.901	28.366	30.612	33.350	38.291	44.335	50.985	57.505	61.656	65.410	69.957	73.166	80.077
46	21.929	25.041	26.657	29.160	31.439	34.215	39.220	45.335	52.056	58.641	62.830	66.617	71.201	74.437	81.400
47	22.610	25.775	27.416	29.956	32.268	35.081	40.149	46.335	53.127	59.774	64.001	67.821	72.443	75.704	82.720
48	23.295	26.511	28.177	30.755	33.098	35.939	41.079	47.335	54.196	60.907	65.171	69.023	73.683	76.969	84.037
49	23.983	27.249	28.941	31.555	33.930	36.818	42.010	48.335	55.265	62.038	66.339	70.222	74.919	78.231	85.351
50	24.674	27.991	29.707	32.357	34.764	37.689	42.942	49.335	56.334	63.167	67.505	71.420	76.154	79.490	86.661