University of Eswatini

Final Examination – 2020/21

Env. Health I

Title of Paper : Algebra for Health Sciences
Course Number : EHS101
Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. Without using a calculator, showing ALL your steps, find the value of

$$3\log 5 + \log 24 - \log 3.$$ [4 marks]

b. Considere the straight line segment from $A(-2,3)$ to $B(4,-5)$.

i. Find the length of the straight line AB [4 marks]

ii. Find the equation of AB in *general form* [6 marks]

c. Given the matrices

$$A = \begin{pmatrix} 2 & -3 \\ 1 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 4 & 1 \end{pmatrix},$$

evaluate

i. $2A - 5B^T$

ii. BA^T

[6 marks]

d. Find the value of the sum

$$\sum_{n=0}^{150} 640\left(\tfrac{9}{8}\right)^n.$$ [6 marks]

e. Use *synthetic division* to find the quotient and remainder of

$$\frac{2x - x^2 + x^4 + 7}{x+2}.$$ [6 marks]

g. Solve for x given

i. $x^2 - 4x + 13 = 0$ [4 marks]

ii. $5^{1-x} = \frac{1}{49}$ (correct to 2 d.p.) [4 marks]

h. Given the vectors

$$\boldsymbol{A} = 2\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}, \quad \boldsymbol{B} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix},$$

find

i. $|2\boldsymbol{A} + \boldsymbol{B}|$ [4 marks]

ii. $\boldsymbol{A} \times \boldsymbol{B}$. [6 marks]

Section B

Answer ANY 2 Questions in this section

B.2 a. Use Cramer's rule to solve the simultaneous system

$$\begin{aligned} 2x - y + z &= -4 \\ x + y - 2z &= 0 \\ 2x + z &= 3. \end{aligned}$$

[15 marks]

b. Find the angle between the vectors

$$\boldsymbol{A} = 2\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}, \quad \boldsymbol{B} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}.$$ [10 marks]

B.3 a. Consider the quadratic function

$$y = x^2 + 2x - 15.$$

i. Find the coordinates of the roots [3 marks]
ii. Find the coordinates of the vertex [3 marks]
iii. Find the coordinates of the y-intercept [1 marks]
iv. Make a sketch of the graph of y [4 marks]

b. A rectangular metal sheet whose length is twice its width is crafted into an open box by cutting out 4×4 centimetre squares from each corner, and folding up the flaps. If the volume of the box is $768\ \text{cm}^3$, find the dimensions of the original metal sheet. [6 marks]

c. For the triangle with vertices $A(-2, 4)$, $B(4, 11)$ and $C(0, -5)$, find

i. the interior angle $\hat{C}$. [3 marks]
ii. the *exact* area of the triangle [5 marks]

B.4 a. The 7th term of an AP is 47. If the 11th term is 75, find the first term and the common difference of the AP. [3 marks]

b. A parent sets up a savings account for their child by making monthly deposits, beginning end of January 2021. The first few deposits are as shown below.

Month	*Jan* 2021	*Feb* 2021	*March* 2021	*April* 2021
Deposit	*E*500	*E*475	*E*451.25	*E*428.69

If the deposits continue in the same trend, find

i. the deposit in June 2022 [3 marks]

ii. the month in which the deposit will be 131.76 [4 marks]

iii. the *total* deposited by December 2023 [5 marks]

iv. the *total* deposited if the deposits continue *"forever"* [3 marks]

c. In the binomial expnasion of

$$\left(x^2 - \frac{2}{x}\right)^{18},$$

find the first 4 terms. [7 marks]

B.5 a. Consider the polynomial

$$P(x) = 2x^3 + Ax^2 + Bx + 6,$$

where A and B are constants.

i. Given that $(x-3)$ is a factor of $P(x)$, while a remainder of 12 is left when $P(x)$ is divided by $(x+1)$, find the values of A and B. [5 marks]

ii. Hence factorise $P(x)$ and find all its roots. [5 marks]

b. Prove that

$$1 - \frac{\sin^2 A}{1 + \cos A} = \cos A. \quad \text{[5 marks]}$$

c. Solve for x :

i. $\log_2(x-3) + \log_2(x+3) = 4$ [5 marks]

ii. $e^{6-x} = 2^x$ [5 marks]

END OF EXAMINATION