University of Eswatini



Final Examination - November 2018

Title of Paper : Algebra for Health Sciences

Course Number : EHS101

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY 2 questions in Section B.

4. Show all your working.

5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A Answer ALL Questions in this section

A.1 a. i. State 3 properties (laws) of logarithms.

[3 marks]

ii. Express the equation

$$4 + \log_2 x = n$$

in exponential form.

[3 marks]

iii. Without using a calculator, evaluate

$$\log 75 + 3\log 2 - \log 6.$$

[5 marks]

b. Evaluate the determinant

$$\begin{vmatrix} -4 & 2 & 3 \\ 5 & -1 & 0 \\ 6 & 9 & -7 \end{vmatrix}.$$
 [6 marks]

c. Evaluate

$$3+4i+\frac{8}{1+i}$$

and leave your answer in the form a + ib.

[5 marks]

d. Find the equation of the straight line passing through (-6, -2) and *perpendicular* to the straight line 4x - 2y = 7.

[7 marks]

e. Find the remainder when $P(x) = x^4 - 2x^2 - 7x + 17$ is divided by

i.
$$x-3$$

[2 marks]

ii.
$$x+2$$

[2 marks]

f. In the binomial expansion of

$$\left(x+\frac{1}{x}\right)^{16}$$
,

find the first 3 terms.

[7 marks]

g. Given the vectors $\mathbf{A} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = -\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ find

i.
$$|A+B|$$

[3 marks]

ii. $A \cdot B$

[3 marks]

h. Given that $\sin \theta = -\frac{5}{13}$ while $\tan \theta > 0$, find the exact values of $\cos \theta$ and $\tan \theta$. [4 marks]

Section B Answer ANY 2 Questions in this section

B.1 a. Given the matrices
$$A=\begin{pmatrix}2&-3&1\\4&0&-2\end{pmatrix}$$
 and $B=\begin{pmatrix}3&-1\\4&2\\-2&1\end{pmatrix}$, work out

- i. $3A^{T} 2B$
- ii. $5A^{T} + 4B^{T}$
- iii. AB iv. A

[7 marks]

b. Given the vectors $\mathbf{A} = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\mathbf{B} = 5\hat{\mathbf{i}} - 12\hat{\mathbf{k}}$, find

i. $A \times B$

[6 marks]

ii. the angle between the vectors \hat{A} and \hat{B} (correct to 1 d.p.)

[6 marks]

c. Prove that

$$\tan A + \frac{\cos A}{1 + \sin A} = \sec A.$$
 [6 marks]

- **B.2** a. Consider the triangle whose vertices are given by A(4,2), B(-4,12) and C(0,-5). Find
 - i. the equation of side AB, expressing it in general form

[4 marks]

ii. the interior angle \hat{A}

[4 marks]

iii. the perimeter of the triangle

[4 marks]

iv. the EXACT area of the triangle

[2 marks]

b. Consider the quadratic function

$$y = 85 - 24x - 4x^2.$$

i. Find the coordinates of the x- and y-intercepts.

[4 marks]

ii. Find the coordinates of the turning point.

[3 marks]

iii. Make a sketch of the graph of the function.

[4 marks]

B.3 a. Solve for x given

i. $7^{2x-5} = 97$ (correct to 2 d.p.)

[3 marks]

ii. $\log_3(x^2+2)=3$

[4 marks]

b. On 01 January 2015, a photocopier is bought valued at E120,000. If it depreciates continuously at 9.8% p.a., its value is given by

$$V(t) = 120\,000e^{-0.098t}.$$

where t is its age in years. Find

i. the book value of the photocopier on 30 June 2018

[2 marks]

ii. the date corresponding to the half-life of the photocopier.

[4 marks]

c. Find the quotient and remainder of

$$\frac{x^4 - 4x^3 + 3x - 11}{x^2 - 2}$$
. [6 marks]

d. Given that both x-2 and x+3 are factors of $P(x)=x^3+Ax^2+Bx-12$, find the values of A and B. [6 marks]

i.
$$\sum_{n=5}^{1000} (8n-3)$$

B.4 a. Find the value of each sum i.
$$\sum_{n=5}^{1000} (8n-3)$$
 ii. $\sum_{n=0}^{500} 50(1.01)^n$ (correct to 1 d.p.)

[3,3 marks]

b. Find the value(s) of x for which the sequence

$$x-1, x+3, 2x^2+1$$

forms an arithmetic progression.

[5 marks]

- c. A child is given one big push on the swing, riding through a 4m swing. If each subsequent swing is 1.5% shorter than the previous, find
 - i. the length of the 50th swing (in cm correct to 2 d.p.) [2 marks]
 - ii. the total distance travelled by the child before the swing stops. [3 marks
- d. In the binomial expansion of

$$\left(x^2 - \frac{y^2}{x^3}\right)^{19},$$

find

i. the 11th term

[3 marks]

ii. the term involving x^{-42}

[6 marks]

END OF EXAMINATION_