University of Swaziland



Re-sit Examination - July 2017

BSc in Environmental Sciences I

Title of Paper : Calculus for Health Sciences

Course Number: EHS102

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY 2 questions in Section B.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A Answer ALL Questions in this section

A.1 Evaluate

i.
$$\lim_{x \to 2} \left(\frac{2x - x^2}{x^2 - 4} \right)$$
 [4 marks]

ii.
$$\lim_{x \to \infty} \left(\frac{2x^2 - 5}{2 + 3x^2 - x^3} \right)$$
 [4 marks]

A.2 a. Use the limit definition to find $\frac{df}{dx}$ given

$$f(x) = x^2.$$
 [10 marks]

b. Find y' if

i.
$$y = x^2 - \frac{2}{x} + 12\sqrt{x}$$
 [4 marks]

ii.
$$y = 30 - e^{2x} + \cos 3x - \ln(x)$$
 [5 marks]

iii.
$$y = (x^2 + 1)^{15}$$
 [5 marks]

c. Find y''' if

$$y = e^{-2x} - \sin 2x.$$
 [6 marks]

A.3 Integrate

i.
$$\int_{1}^{9} \left(\frac{7}{\sqrt{x}} - 2x + 3x^{2} \right) dx$$
 [6 marks]

ii.
$$\int_1^9 \left(\frac{10}{x} + e^x - \frac{5}{x^2}\right) dx$$
 (correct to 2 d.p.) [6 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Consider the graph of the function

$$y = \frac{x}{8 - 3x}.$$

i. Find the value(s) of x when y = 1.

[4 marks]

ii. Hence, find the equation of the tangent(s) and normal(s) at y = 1.

[9 marks]

b. A retail shop has determined that the price demand function for its product is given by

$$p = 15 - 0.0025x$$

where p is the unit selling price and x is the number of units sold per month. If each unit costs E3 while fixed monthly costs stand at E5,500, find

i. the number of units per month that yield maximum profit

[9 marks]

ii. the maximum profit

[3 marks]

B.2 a. Differentiate and simplify

i.
$$F(x) = (x^2 - 2x + 1)e^{2x}$$

[7 marks]

ii.
$$G(x) = \ln\left(\frac{x}{x+1}\right)$$

[6 marks]

b. Consider the function

$$f(x) = 12 + 12x - x^3.$$

i. Find the stationary points of f(x) and their nature.

[6 marks]

ii. Find the inflexion points of f(x)

[2 marks]

iii. Make a sketch of the graph of y = f(x).

[4 marks]

B.3 a. Evaluate

i.
$$\int_{1}^{4} \frac{\mathrm{d}x}{x(x+1)}$$
 [10 marks]

ii.
$$\int_{0}^{\sqrt{3}} \frac{2x \, dx}{\sqrt{x^2 + 1}}$$
 [7 marks]

b. After the launch of a new video game, the rate at which the sales grow is given by

$$S'(t) = \frac{12}{\sqrt{1+4t}},$$

in thousands of units per month, where t is the number of months after the launch. Find the total number of units sold in

- i. the first 6 months
- ii. the second half of the year

[8 marks]

- **B.4** a. A car is travelling at 108km/h. At a certain instant the brakes are applied producing a constant deceleration of 0.8 m/s². find
 - i. the formula for the velocity of the car during the deceleration [4 marks]
 - ii. the formula for the height of the car during the deceleration [4 marks]
 - iii. the time taken before the car stops

[3 marks]

iv. the distance covered during the deceleration

[3 marks]

b. Find the area of the region bounded by the curves $y = x^2 - 4x$ and $y = 8x - x^2$. [10 marks]

END OF EXAMINATION $oldsymbol{\bot}$