University of Swaziland

Final Examination – December 2016

BSc in Environmental Sciences I

Title of Paper	:	Algebra for Health Sciences
Course Number	:	EHS101
Time Allowed	:	Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. Find the value of the sum

i. $\sum_{n=-5}^{75}(5-8n)$ [5 marks]

ii. $\sum_{n=0}^{\infty}75\left(\frac{4}{9}\right)^n$ [4 marks]

b. Given that $\tan\theta = \frac{4}{3}$ while $\sin\theta < 0$, find the *exact* value of $\cos\theta$. [5 marks]

c. Prove that

$$\cos^2\theta\big(\sin^2\theta + \cos^2\theta + \tan^2\theta\big) = 1.$$ [5 marks]

c. Evaluate the complex number

i. $(3-5i)^2 - (3i+5)^2$ [4 marks]

ii. $\dfrac{4i+3}{4-3i}$ [4 marks]

and leave your answer in the form $a + ib$.

d. Find the equation of the straight line from $(4, -5)$ to $(-3, 9)$. [6 marks]

e. Solve for x (express non-exact answers correct to 2 d.p.)

i. $3^{2x-1} = 7439$ [4 marks]

ii. $\ln\left(\dfrac{4x-7}{2x+15}\right) = 0$ [5 marks]

f. Given the vectors $\boldsymbol{A} = 12\hat{\imath} - 16\hat{k}$ and $\boldsymbol{B} = 8\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, find

i. $\boldsymbol{A}\cdot\boldsymbol{B}$ [2 marks]

ii. $\boldsymbol{A}\times\boldsymbol{B}$ [6 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Evaluate

$$\begin{vmatrix} 4 & 0 & -2 & 3 \\ 1 & 0 & 0 & -4 \\ 0 & -3 & 5 & 2 \\ 7 & 0 & 0 & 3 \end{vmatrix}.$$

[10 marks]

b. Use Cramer's rule to solve

$$\begin{aligned} 2x - y + 3z &= 0 \\ x + 2y - 2z &= 6 \\ 5x + 2y &= 12 \end{aligned}$$

[15 marks]

B.2 a. Consider the triangle with vertices $A(4,7)$, $B(-5,2)$ and $C(6,-9)$. Find

i. the perimeter of the triangle [6 marks]

ii. the interior angle $\hat{A}$ [4 marks]

iii. the area of the triangle [6 marks]

b. A circle is centred at $C(-4,7)$ and passes through the point $(-5,2)$. Find

i. the equation of the circle in *general form* [5 marks]

ii. the perimeter and area of the circle [2,2 marks]

B.3 a. In the binomial expansion of

$$\left(x^2 - \frac{1}{x^3}\right)^{18}$$

find

i. the 17th term [3 marks]

ii. the term involving x^{-4} [7 marks]

b. Consider the polynomial

$$P(x) = 12x^3 + Ax^2 - 17x - 10,$$

where A is a constant. It is given that $x + 1$ is a factor of $P(x)$.

i. Find the the value of A [3 marks]

ii. Hence, or otherwise, factorise $P(x)$ and determine its roots. [7 marks]

c. Use *synthetic division* to find the quotient and remainder of

$$\frac{x^4 - 2x^3 + 2x - 7}{x + 2}.$$ [5 marks]

B.4 a. Solve for x (expressing non-exact aswers correct to 2 d.p.)

i. $3 \cdot e^{x-2} = 7^x$ [6 marks]

ii. $\log_4(5x + 1) - \log_4(x + 7) = 1$ [6 marks]

b. On 01 January 2016, a sum of E7,500 is invested in an account which pays 9% interest, compounded daily. The amount grows according to the formula

$$A(t) = 7,500\left(1 + \frac{0.09}{365}\right)^{365t},$$

where t is the number of years after 01 January 2016. Find the

i. amount in the account on 01 July 2020 [2 marks]

ii. date on which the amount in the account will reach E14,000. [6 marks]

c. The pH of a aqueous solution is given by pH $= -\log\left[H^+\right]$ where $\left[H^+\right]$ is the concentration of hydronium ions in the solution.

i. Find the pH correct to 2 decimal places for lemon juice with $\left[H^+\right] = 8.46 \times 10^{-4}$M [2 marks]

ii. Find the concentration of hydronium ions correct to 3 significant figures for egg white with pH $= 8.27$ [3 marks]