University of Swaziland

Final Examination – November 2014

BSc in Environmental Sciences I

Title of Paper : Algebra for Health Sciences
Course Number : EHM106
Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. Evaluate

$$\sum_{n=0}^{70}(4n+3).$$ [10 marks]

b. Expand using the binomial theorem

$$\left(2x^2 - \frac{1}{x}\right)^5.$$ [10 marks]

c. Use *synthetic division* to evaluate

$$\frac{x^4 - x^3 + 2x - 7}{x-2}.$$ [5 marks]

d. Find the length of the straight line AB between $A(-3, 2)$ and $B(1, -6)$. [5 marks]

e. Solve for x given

i. $\log_7(10x - 1) = 2$ [5 marks]

ii. $3^{x+1} = 70$ [5 marks]

f. Given the matrices $A = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$, find the value of

i. $A^T B$ [5 marks]

ii. $B^T A$ [5 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Use Cramer's rule to solve

$$\begin{aligned} 3x - 2y + z &= 13 \\ 2x + y - 3z &= -2 \\ x - 7y &= 23. \end{aligned}$$

[18 marks]

b. Find the value(s) of the scalar a such that the vectors $\underline{A} = 4a\hat{\imath} + 2a\hat{\jmath} + 3\hat{k}$ and $\underline{B} = 3a\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ are perpendicular. [7 marks]

B.2 a. Find the value of

i. $20 + 25 + 30 + \cdots + 2,000$ [5 marks]

ii. $\sum_{n=0}^{\infty} 30\left(\frac{2}{5}\right)^n$ [5 marks]

b. A child on a swing is given one big push, riding through a 5 metre arc. If the lengths of successive swings decrease by 5%, find the total distance travelled by the child as the swing comes to a stop. [3 marks]

c. Prove that

$$1 - \frac{\cos^2 A}{1 + \sin A} = \sin A.$$

[8 marks]

d. Find the general solution of

$$\cos(\theta - 10^0) = -\tfrac{1}{2}.$$

[4 marks]

B.3

a. Work out and express in the form $a + ib$.

i. $2i(2+3i) - 3i(3-2i)$ [3 marks]

ii. $\dfrac{3-4i}{4+3i}$ [4 marks]

iii. $(\sqrt{3} - i)^6$ (using de Moivre's theorem) [6 marks]

b. Given that $x + 3$ is a factor of the polynomial $P(x) = x^3 + Ax^2 + Bx - 6$, while dividing $P(x)$ by $x - 1$ leaves a remainder of -8, find the values of A and B. [6 marks]

c. Find the coordinates of the centre and radius of the circle with equation

$$2x^2 + 2y^2 - 10x + 6y - 183 = 0.$$ [6 marks]

B.4

a. Solve

$$\log_5 x + \log_5(x-4) = 1.$$ [8 marks]

b. In a college campus with a population of 2,000, the number of people that have heard a rumour is given by

$$P(t) = 2,000\left(1 - e^{-0.09t}\right),$$

where t is the number of days after the rumour has began. Find the

i. number of people that have heard the rumour after 2 days [2 marks]

ii. number of days it takes for 50% of college to hear the rumour. [5 marks]

c. For the binomial expansion of

$$\left(x + \frac{y}{x}\right)^{24},$$

find the

i. first 4 terms [6 marks]

ii. the 19th term [4 marks]