UNIVERSITY OF SWAZILAND Faculty of Health Sciences

DEGREE IN ENVIRONMENTAL HEALTH

MAIN EXAMINATION PAPER 2013

TITLE OF PAPER

: CLCULUS FOR HEALTH SCIENCES

COURSE CODE

EHM 107

DURATION

2 HOURS

MARKS

100 MARK

INSTRUCTIONS

READ THE QUESTIONS & INSTRUCTIONS

CAREFULLY

ANSWER ANY FOUR QUESTIONS

EACH QUESTION CARRIES 25 MARKS

: WRITE NEATLY & CLEARLY

NO PAPER SHOULD BE BROUGHT INTO NOR OUT

OF THE EXAMINATION ROOM

BEGIN EACH QUESTION ON A SEPARATE SHEET

OF PAPER

DO NOT OPEN THIS QUESTION PAPER UNTIL PERMISSION IS GRANTED BY THE INVIGILATOR

Question 1

(a) Evaluate

(i)
$$\lim_{x \to 1} \frac{x^2 - x}{x^2 - 1}$$

[6 marks]

(ii)
$$\lim_{x \to \infty} \frac{x^2 - x}{x^2 - 1}$$

[6 marks]

(b) Use the *limit definition* of the derivative to find f'(x)given

$$f(x) = 2x - x^2.$$

[13 marks]

Question 2

(a) Differentiate

(i)
$$y = 2^{-x}$$

[5 marks]

(i)
$$y = 2^{-x}$$

(ii) $F(x) = (1 + 4x)^{15}$

[5 marks]

(iii)
$$H(x) = \ln(\sec x + \tan x)$$

[5 marks]

(b) Integrate

$$\int 24x^2 e^{-x} \, \mathrm{d}x$$

[10 marks].

Question 3

- (a) Find y''' for the function $y = e^{-2x}$. [6 marks]
- (b) Evaluate find

(i)
$$\int \left(4x + \frac{1}{x} - \frac{6}{x^2}\right) dx$$
 [3 marks]

(ii)
$$\int (1 + e^x - \cos x) dx$$
 [4 marks]

(iii)
$$\int_{1}^{9} \left(6X^{\frac{1}{2}} - X^{-\frac{1}{2}}\right) dX$$
 [6 marks]

(iv)
$$\int \frac{x}{x+2} dx$$
 [6 marks]

Question 4

(a) Differentiate

(i)
$$y = \ln x - \cos 2x + e^{-4x} - \frac{5}{x^2}$$
 [4 marks]

(ii)
$$y = \frac{x^2}{2 + x^2}$$
 [7 marks]

- (b) A bottling company has assigned you to design a closed cylindrical container with a capacity of 16π cubic centimetres.
 - (i) Show that its external surface area S is given by

$$S(r) = 2\pi r^2 + \frac{32\pi}{r}$$

where r is the radius.

[5 marks]

(ii) Find the value of r for which the surface area is minimum. [7 marks]

(iii) Hence find the dimensions of the cylinder that utilizes the least amount of material. [2 marks]

Question 5

(a) Evaluate

(i)
$$\int \left(1 - 2x^2\right)^2 dx$$
 [4 marks]
(ii)
$$\int \frac{12x dx}{\sqrt{1-x^2}}$$
 [7 marks]

(b) Find the *exact* area of the region enclosed by the curves $y = x^2 - 2$ and $y = 2 - x^2$. [16 marks]

Question 6

- (a) Find y^{iv} for the function $y = 80\sqrt{x} \frac{1}{x}$. [6 marks]
- (b) Consider the function

$$f(x) = x^3 - 12x + 2.$$

Locate the stationary points of f(x) and classify it them. [9 marks]

(c) Use partial fractions to evaluate

$$\int \frac{x}{(x+1)(x-2)} dx.$$
 [10 marks]