University of Swaziland



Final Examination May 2012

Diploma in Environmental Health Sciences I

Title of Paper

: Calculus for Health Sciences

Course Number

: EHM107

Time Allowed

: Two (2) hours

Instructions

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- 1. This paper consists of SIX questions.
- 2. Each question is worth 25%.
- 3. Answer ANY FOUR questions.
- 4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

(a) Evaluate

(i)
$$\lim_{t \to 1} \frac{t^2 - 1}{t + 1}$$
 [4 marks]

(ii)
$$\lim_{x \to 0} \frac{1 + \sin x}{1 + \cos x}$$
 [3 marks]

(b) Differentiate

(i)
$$H(v) = \frac{v^5 - 2v^3 + 3}{v^2}$$
 [4 marks]
(ii) $F(u) = \frac{u}{u - 1}$ [7 marks]

(ii)
$$F(u) = \frac{u}{u-1}$$
 [7 marks]

(c) Integrate

$$\int_{1}^{e} \frac{x+2}{x} \, \mathrm{d}x.$$
 [7 marks]

Question 2

(a) Differentiate

(i)
$$H(x) = 3x^3 - \frac{1}{3x^3} - 7$$
 [4 marks]

(ii)
$$F(x) = \ln \pi + \tan x - e^{-2x}$$
 [4 marks]

(b) Use the limit definition to find f'(x) given

$$f(x) = 2x - 5x^2$$
. [8 marks]

(c) Evaluate

$$\int \frac{6x}{\sqrt{x^2 + 2}} dx.$$
 [9 marks]

(a) For the function

$$y = x^3 + 3x^2 - 9x + 2,$$

find

i. the intervals in which the graph is increasing/decreasing [3 marks]

ii. intervals in which the graph is concave up/down

[3 marks]

iii. stationary points and classify them [3 marks]

iv. inflexion points [3 marks]

Hence make a sketch of the graph of y. [5 marks]

(b) Integrate

i.
$$\int \left(\frac{3}{x} - \frac{3}{x^3} - 6e^{-2x}\right) dx$$
 [4 marks]

ii.
$$\int_{-2}^{2} t \left(t^3 - \frac{2}{t^3} \right) dt$$
 [4 marks]

(a) Integrate

i.
$$\int_0^1 \left(3x^2 - 4x + 6\sqrt{x}\right) dx$$
 [4 marks]

ii.
$$\int (x+3)^2 dx$$
 [4 marks]

(b) A farmer needs to construct a rectangular holding for his livestock. On one of the sides, he needs to use heavy-duty fence which costs E 50 per metre. Regular fencing, to be used on the other sides costs E 30 per metre. If his budget for the job is E 24,000.00, find the dimensions of the largest holding he can construct.

[17 marks]

Question 5

(a) Find the indicated derivatives:

ii.
$$H(v) = (v-1)e^{2v}$$
, H''' [8 marks]

(b) Evaluate

$$\lim_{x \to \infty} \frac{2x^2 + 3x - 6}{5 - 4x - x^2}.$$

[5 marks]

(c) Find the area of the region enclosed by the curves $y = x^2$ and y = 2x + 8. [10 marks]

(a) Differentiate and simplify

i.
$$G(\lambda) = \lambda e^{\lambda} - e^{\lambda} + 2$$
 ii.
$$y = \frac{\sin x}{1 + \cos x}$$

[3 marks]

ii.
$$y = \frac{\sin x}{1 + \cos x}$$

[6 marks]

(b) Integrate

$$\int x^2 \sin x \, \mathrm{d}x$$

[8 marks]

(c) Use partial fractions to integrate

$$\int \frac{x+2}{x^2-x-6} \, \mathrm{d}x.$$

[8 marks]