

## UNIVERSITY OF SWAZILAND



### Supplementary Examination 2005

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<b>Title of Paper</b>	:	Health Sciences Mathematics
<b>Program</b>	:	Dip./BSc. Env. Health
<b>Course Number</b>	:	HSM 110
<b>Time Allowed</b>	:	Three (3) Hours
<b>Instructions</b>	:	<ol style="list-style-type: none"><li>1. This paper consists of EIGHT questions on TWO pages.</li><li>2. Answer any five (5) questions.</li><li>3. Non-programmable calculators may be used.</li></ol>
<b>Special Requirements:</b>		None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION  
TO DO SO IS GRANTED BY THE INVIGILATOR.

**Question 1**

(a) Find the quotient  $Q(x)$  and remainder  $R(x)$  when  $P(x) = x^6 - x^4 + x^3 - x + 1$  is divided by  $D(x) = x^2 + 1$ .

[10 marks]

(b) Evaluate the following limits:

(i)  $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 7}{x^2 - 3x + 2}$

(ii)  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

[10 marks]

**Question 2**

(a) Let  $A$  be an angle in  $Q-IV$  (fourth quadrant). Give exact values of  $\tan A$  and  $\cos 2A$ , given that

$$\sin A = -\frac{3}{5}.$$

[10 marks]

(b) Find the middle term in the expansion of  $\left(y - \frac{x}{y^2}\right)^{12}$ .

[10 marks]

**Question 3**

(a) Prove the trigonometric identity:  $\frac{2 \tan x}{1 + \tan^2 x} = 2 \sin x \cos x$ .

[10 marks]

(b) Find all roots of the polynomial equation  $x^3 - 2x^2 - x + 2 = 0$ .

[10 marks]

**Question 4**

(a) Convert to a sum/difference:  $\sin 80^\circ \sin 20^\circ$ .

[8 marks]

(b) For the function, find and classify all stationary points, determine intervals of increase and decrease, and sketch its graph:  $y = x^3 - 3x + 1$ .

[12 marks]

**Question 5**

(a) Find the equation of the line tangent to  $y = x^2 - \sqrt{x}$  at  $(4, 14)$ .

[10 marks]

Q5 (b) Use the binomial series to evaluate, correct to 4 decimal places:  $\sqrt[4]{82}$ .

[10 marks]

### Question 6

(a) Differentiate the following:  $y = \frac{x^2 + 1}{x^2 - 1}$ .

[5 marks]

(b) Use EITHER Gaussian Elimination OR Cramer's rule to solve the following linear system of equations:

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 7 \\ x_1 + 5x_2 + 7x_3 = 20 \\ -3x_1 + x_2 - 5x_3 = -12 \end{cases}$$

[10 marks]

(c) Find value(s) of  $c$  such that the vectors  $a = (c, -2, 1)$  and  $b = (2c, c, -4)$  are orthogonal.

[5 marks]

### Question 7

(a) Use the definition of the derivative to find  $f'(x)$ , given that  $f(x) = x^2 - x$ .

[10 marks]

(b) A ladder 10 m long leans against a vertical wall. If the bottom of the ladder is being pulled away from the wall at a rate of 50 cm/s, how quickly is the top of the ladder descending down the wall when the base of the ladder is 4 m from the wall?

[10 marks]

### Question 8

(a) For the following function, evaluate  $y''$ :

$$y = \sqrt{2x-1}.$$

[5 marks]

(b) Differentiate the following:

$$y = e^{2x} \ln(x^2 - 2x).$$

[5 marks]

(c) A new property owner has 800 m of fencing with which to fence off his rectangular plot of land. If the shape of the plot is rectangular, and three sides are to be fenced (as neighbours already have a fence bordering the third side), determine the dimensions of the field of maximum area that can be enclosed by the fencing available.

[10 marks]

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