UNIVERSITY OF SWAZILAND
FACULTY OF EDUCATION
SUPPLEMENTARY EXAMINATION PAPER 2010

TITLE OF PAPER: CURRICULUM STUDIES IN MATHEMATICS

COURSE CODE: EDC 281

PROGRAMME: B.ED 2 & PGCE

TIME ALLOWED: THREE (3) HOURS

TOTAL MARKS: 100

APPENDICES: TRIGONOMETRY & SYMMETRY CHAPTERS

INSTRUCTIONS: ANSWER ANY **FOUR** QUESTIONS. EACH QUESTION IS WORTH 25 MARKS.

This paper contains 2 pages including this one

Question 1

(a) What is a concept? [2]
(b) State the **two** principles of concept development [4]
(c) Name **four** sources of misconceptions [4]
(d) Discuss tension as a strategy for motivation.[15]

Question 2

Refer to appendix 1 and sections 14.1-14.7 of the SGCSE syllabus to answer (a) and (b)

(a) Write a behavioural objective, for **one** section of your choice, such that the objective includes all that behaviourists recommend for stating objectives. Point out each part to the reader. [5]
(b) Prepare a concept map for "trigonometry" in the syllabus. [20]

Syllabus Extract

14. Trigonometry [Topic Area: Shape, Position and Space]	
All learners should be able to: 14.1 Apply Pythagoras Theorem. 14.2 Calculate sides and angles of a right-angled triangle using sine, cosine and tangent ratios. 14.3 Solve simple problems involving angles of depression and elevation (from right-angled triangles)	14.4 Find sine and cosine for obtuse angles 14.5 Use sine rule and the cosine formula for trigonometric problems in 2-dimensions. 14.6 Use the formula A = ½absinC for the area of triangle ABC 14.7 Use trigonometric problems in 3-dimensions including angle between a line and a plane

Question 3

Using the Realistic Mathematics Education, (RME), philosophy prepare a discussion lesson, for 6 groups of 8, on line symmetry [25]. Use appendix 2.
Syllabus Objective: All learners should be able to recognise line symmetry in 2-dimensions.

Question 4

(a) With the help of a diagram explain what it means to say Bloom's taxonomy for the cognitive domain is hierarchical in nature. [10]
(b) Name and briefly explain each part of higher order abilities in Bloom's taxonomy. [6]
(c) Explain the following statement "An item is said to be testing higher order abilities dependent on what was done in the class during instruction."[9]

Question 5

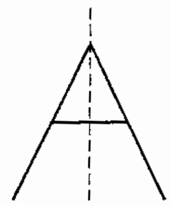
Write an essay entitled "The injustice of a differentiated curriculum: The case of core and extended mathematics." [25]

APPENDIX 1

4.3 Symmetry

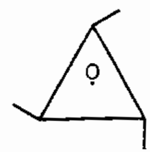
Line symmetry

The letter A has one line of symmetry, shown dotted.



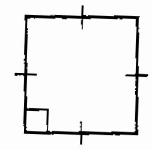
Rotational symmetry

The shape may be turned about O into three identical positions. It has rotational symmetry of order 3.

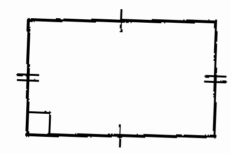


Quadrilaterals

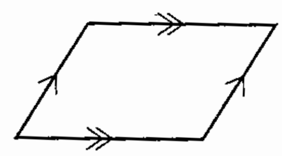
1. *Square*
all sides are equal, all angles 90°, opposite sides parallel; diagonals bisect at right angles.



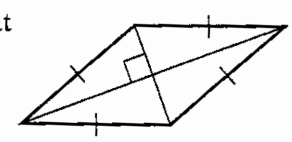
2. *Rectangle*
opposite sides parallel and equal, all angles 90°, diagonals bisect each other.



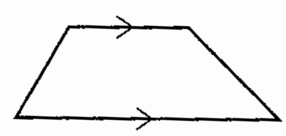
3. *Parallelogram*
opposite sides parallel and equal, opposite angles equal, diagonals bisect each other (but not equal).



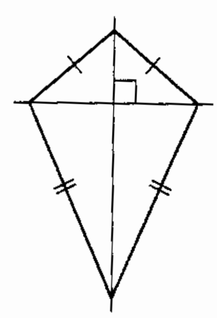
4. *Rhombus*
a parallelogram with all sides equal, diagonals bisect each other at right angles and bisect angles.



5. *Trapezium*
one pair of sides is parallel.



6. *Kite*
two pairs of adjacent sides equal, diagonals meet at right angles bisecting one of them.

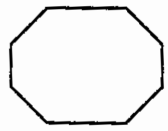


Exercise 5

1. For each shape state:
   (a) the number of lines of symmetry (b) the order of rotational symmetry.

(a)

(b)

(c)

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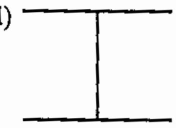
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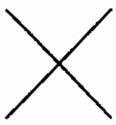
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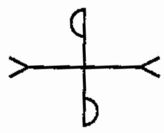
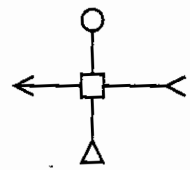
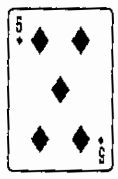

(g)

(h)

(i)

(j)

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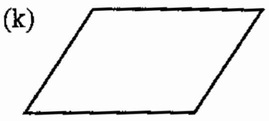
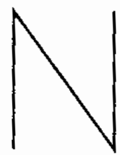
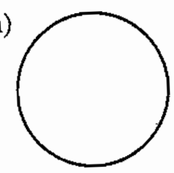
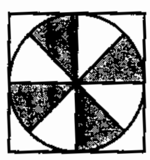
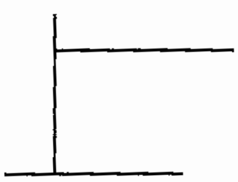

(l)

(m)

(n)


2. Add one line to each of the diagrams below so that the resulting figure has rotational symmetry but not line symmetry.

(a)

(b)


3. Draw a hexagon with just two lines of symmetry.

4. For each of the following shapes, find:
   (a) the number of lines of symmetry
   (b) the order of rotational symmetry.

   square; rectangle; parallelogram; rhombus; trapezium; kite; equilateral triangle; regular hexagon.

In questions **5** to **15**, begin by drawing a diagram.

5. In a rectangle KLMN, $L\hat{N}M = 34°$. Calculate:
   (a) $K\hat{L}N$ (b) $K\hat{M}L$

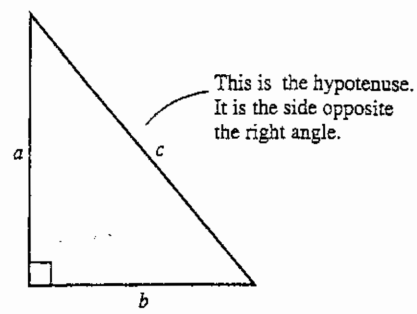
6. In a trapezium ABCD; $A\hat{B}D = 35°$, $B\hat{A}D = 110°$ and AB is parallel to DC. Calculate:
   (a) $A\hat{D}B$ (b) $B\hat{D}C$

APPENDIX 2

4.2 Pythagoras' theorem

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

$a^2 + b^2 = c^2$



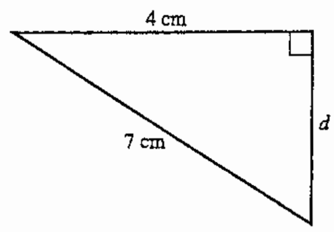
Example

Find the side marked d.

$d^2 + 4^2 = 7^2$

$d^2 = 49 - 16$

$d = \sqrt{33} = 5{\cdot}74\,\text{cm}$ (3 sig. fig.)

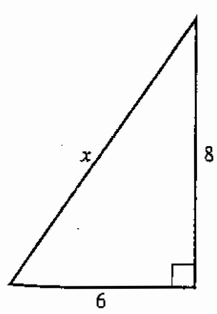


The *converse* is also true:
'If the square on one side of a triangle is equal to the sum of the squares on the other two sides, then the triangle is right-angled.'

Exercise 4

In questions **1** to **10**, find x. All the lengths are in cm.

1.



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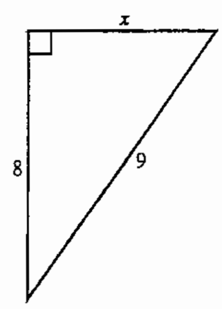

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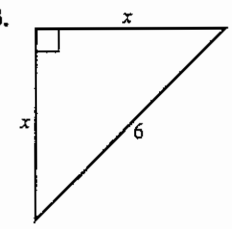
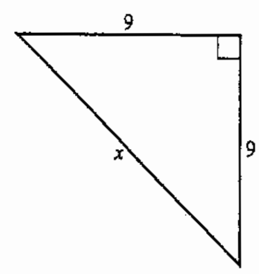
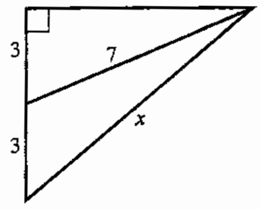


4.



5.



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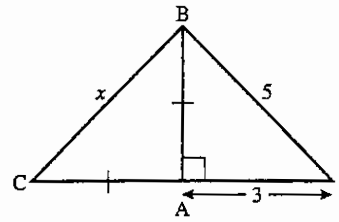

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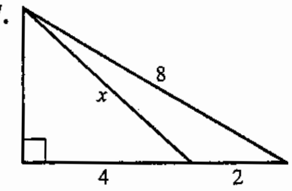

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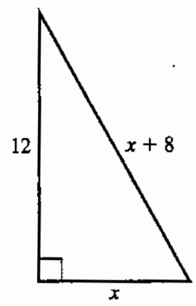
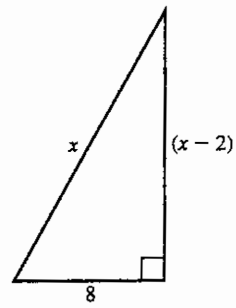
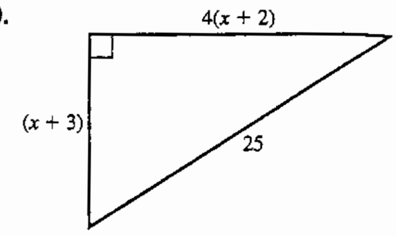


9.



10.



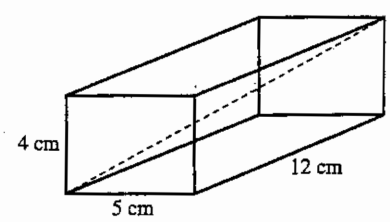
11. Find the length of a diagonal of a rectangle of length 9 cm and width 4 cm.

12. A square has diagonals of length 10 cm. Find the sides of the square.

13. A 4 m ladder rests against a vertical wall with its foot 2 m from the wall. How far up the wall does the ladder reach?

14. A ship sails 20 km due North and then 35 km due East. How far is it from its starting point?

15. Find the length of a diagonal of a rectangular box of length 12 cm, width 5 cm and height 4 cm.



16. Find the length of a diagonal of a rectangular room of length 5 m, width 3 m and height 2·5 m.

17. Find the height of a rectangular box of length 8 cm, width 6 cm where the length of a diagonal is 11 cm.

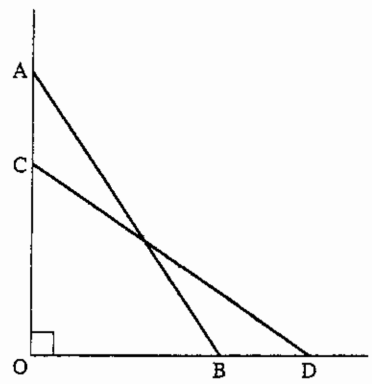
18. An aircraft flies equal distances South-East and then South-West to finish 120 km due South of its starting-point. How long is each part of its journey?

19. The diagonal of a rectangle exceeds the length by 2 cm. If the width of the rectangle is 10 cm, find the length.

20. A cone has base radius 5 cm and *slant* height 11 cm. Find its vertical height.

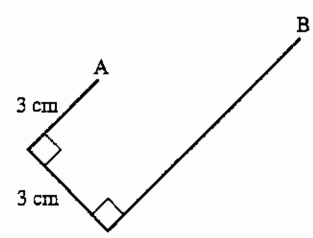
21. It is possible to find the sides of a right-angled triangle, with lengths which are whole numbers, by substituting different values of x into the expressions:
(a) $2x^2 + 2x + 1$ (b) $2x^2 + 2x$ (c) $2x + 1$
((a) represents the hypotenuse, (b) and (c) the other two sides.)
(i) Find the sides of the triangles when $x = 1, 2, 3, 4$ and 5.
(ii) Confirm that $(2x + 1)^2 + (2x^2 + 2x)^2 = (2x^2 + 2x + 1)^2$

22. The diagram represents the starting position (AB) and the finishing position (CD) of a ladder as it slips. The ladder is leaning against a vertical wall.



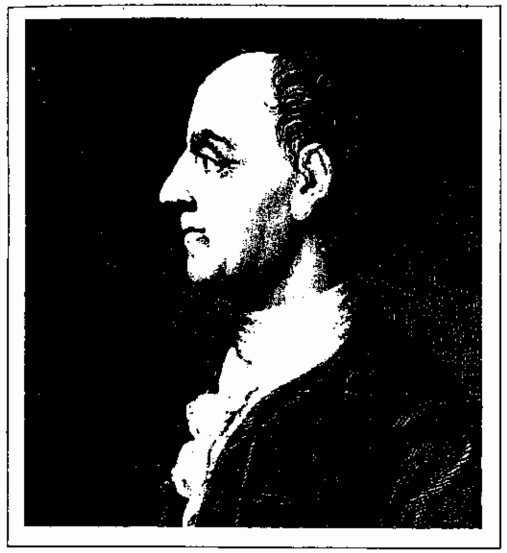
Given: AC = x, OC = 4AC, BD = 2AC and OB = 5 m.
Form an equation in x, find x and hence find the length of the ladder.

23. A thin wire of length 18 cm is bent into the shape shown.
Calculate the length from A to B.



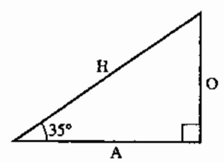
24. An aircraft is vertically above a point which is 10 km West and 15 km North of a control tower. If the aircraft is 4000 m above the ground, how far is it from the control tower?

6 Trigonometry



Leonard Euler (1707–1783) was born near Basel in Switzerland but moved to St Petersburg in Russia and later to Berlin. He had an amazing facility for figures but delighted in speculating in the realms of pure intellect. In trigonometry he introduced the use of small letters for the sides and capitals for the angles of a triangle. He also wrote r, R and s for the radius of the inscribed and of the circumscribed circles and the semi-perimeter, giving the beautiful formula $4rRs = abc$.

6.1 Right-angled triangles



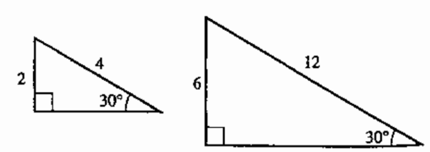
The side opposite the right angle is called the hypotenuse (we will use H). It is the longest side.

The side opposite the marked angle of 35° is called the opposite (we will use O).

The other side is called the adjacent (we will use A).

Consider two triangles, one of which is an enlargement of the other.

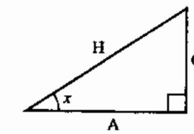
It is clear that the *ratio* $\frac{O}{H}$ will be the same in both triangles.



Sine, cosine and tangent

Three important functions are defined as follows:

$$\sin x = \frac{O}{H}$$
$$\cos x = \frac{A}{H}$$
$$\tan x = \frac{O}{A}$$



It is important to get the letters in the right order. Some people find a simple sentence helpful when the first letters of each word describe sine, cosine or tangent and Hypotenuse, Opposite and Adjacent. An example is:

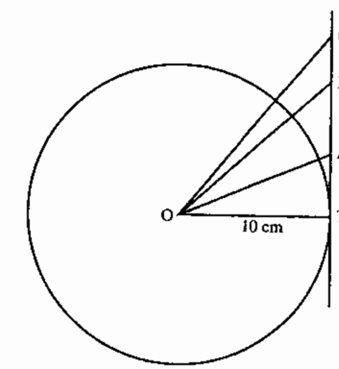
Silly Old Harry Caught A Herring Trawling Off Afghanistan.

e.g. $\text{S O H}: \sin = \frac{O}{H}$

For any angle x the values for $\sin x$, $\cos x$ and $\tan x$ can be found using either a calculator or tables.

Exercise 1

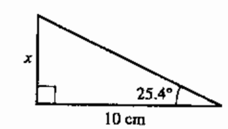
1. Draw a circle of radius 10 cm and construct a tangent to touch the circle at T.
   Draw OA, OB and OC where $A\hat{O}T = 20°$
   $B\hat{O}T = 40°$
   $C\hat{O}T = 50°$
   Measure the length AT and compare it with the value for tan 20° given on a calculator or in tables. Repeat for BT, CT and for other angles of your own choice.



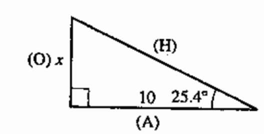
Finding the length of a side

Example 1

Find the side marked x.



(a) Label the sides of the triangle H, O, A (in brackets).



(b) In this example, we know nothing about H so we need the function involving O and A.

$$\tan 25{\cdot}4° = \frac{O}{A} = \frac{x}{10}$$

(c) Find tan 25·4° from tables.

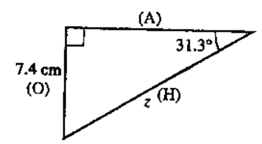
$$0{\cdot}4748 = \frac{x}{10}$$

(d) Solve for x.
$x = 10 \times 0{\cdot}4748 = 4{\cdot}748$
$x = 4{\cdot}75$ cm (3 significant figures)

Example 2

Find the side marked z.

(a) Label H, O, A.



(b) $\sin 31{\cdot}3° = \frac{O}{H} = \frac{7{\cdot}4}{z}$

(c) Multiply by z.
$z \times (\sin 31{\cdot}3°) = 7{\cdot}4$

$$z = \frac{7{\cdot}4}{\sin 31{\cdot}3}$$

(d) On a calculator, press the keys as follows:

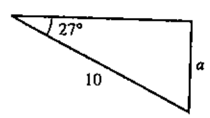
7.4 | ÷ | 31·3 | sin | =

$z = 14{\cdot}2$ cm (to 3 s.f.)

On some calculators you press: 7.4 | ÷ | sin | 31·3 | =

Exercise 2

In questions **1** to **22** all lengths are in centimetres. Find the sides marked with letters. Give your answers to three significant figures.

1.


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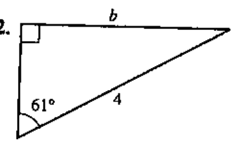

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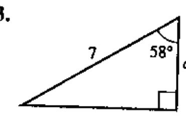

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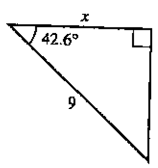

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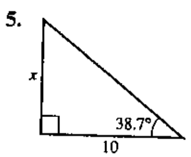

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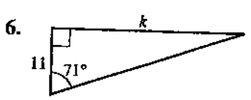

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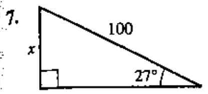
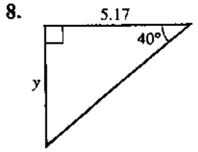


8.


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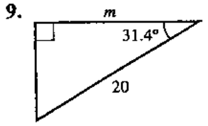

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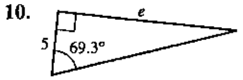

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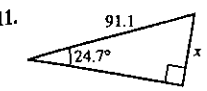

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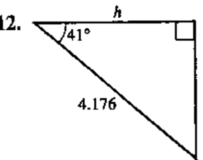

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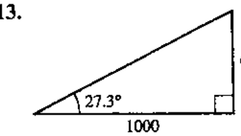

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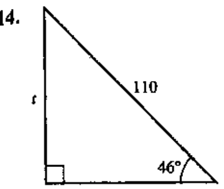

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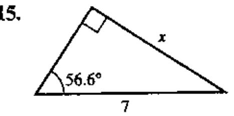

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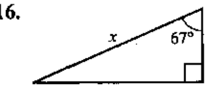

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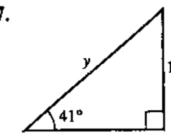

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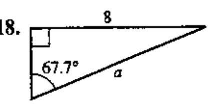

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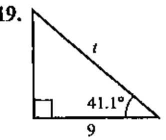

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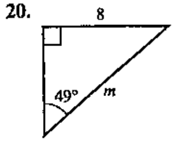

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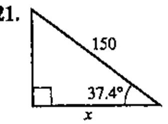

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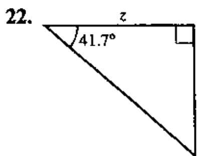
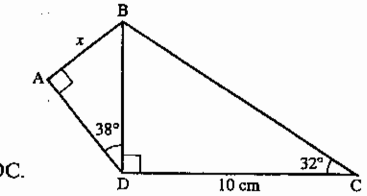


In questions **23** to **34**, the triangle has a right angle at the middle letter.

23. In $\triangle ABC$, $\hat{C} = 40°$, BC = 4 cm. Find AB.
24. In $\triangle DEF$, $\hat{F} = 35{\cdot}3°$, DF = 7 cm. Find ED.
25. In $\triangle GHI$, $\hat{I} = 70°$, GI = 12 m. Find HI.
26. In $\triangle JKL$, $\hat{L} = 55°$, KL = 8·21 m. Find JK.
27. In $\triangle MNO$, $\hat{M} = 42{\cdot}6°$, MO = 14 cm. Find ON.
28. In $\triangle PQR$, $\hat{P} = 28°$, PQ = 5·071 m. Find PR.
29. In $\triangle STU$, $\hat{S} = 39°$, TU = 6 cm. Find SU.
30. In $\triangle VWX$, $\hat{X} = 17°$, WV = 30·7 m. Find WX.
31. In $\triangle ABC$, $\hat{A} = 14{\cdot}3°$, BC = 14 m. Find AC.
32. In $\triangle KLM$, $\hat{K} = 72{\cdot}8°$, KL = 5·04 cm. Find LM.
33. In $\triangle PQR$, $\hat{R} = 31{\cdot}7°$, QR = 0·81 cm. Find PR.
34. In $\triangle XYZ$, $\hat{X} = 81{\cdot}07°$, YZ = 52·6 m. Find XY.

Example

Find the length marked x.



(a) Find BD from triangle BDC.

$$\tan 32° = \frac{BD}{10}$$

$\therefore \quad BD = 10 \times \tan 32°$ ...[1]

(b) Now find x from triangle ABD.

$$\sin 38° = \frac{x}{BD}$$

$\therefore \quad x = BD \times \sin 38°$

$x = 10 \times \tan 32° \times \sin 38°$ (from [1])

$x = 3{\cdot}85$ cm (to 3 s.f.)

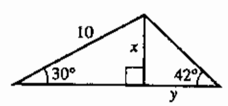
Notice that BD was *not* calculated in [1].

It is better to do all the multiplications at one time.

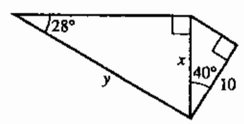
Exercise 3

In questions **1** to **10**, find each side marked with a letter. All lengths are in centimetres.

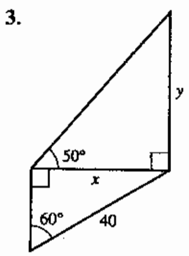
1.



2.



3.



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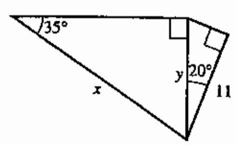
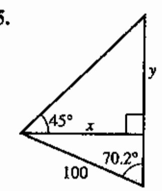


5.



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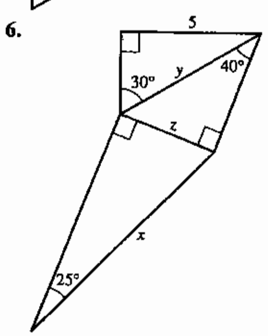

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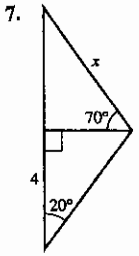

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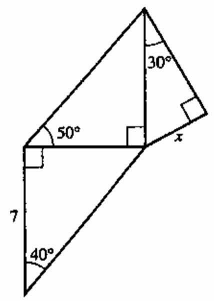

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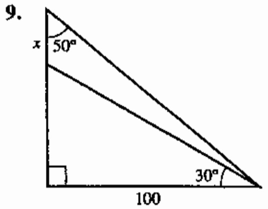
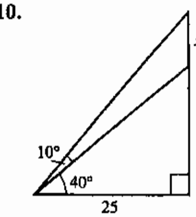
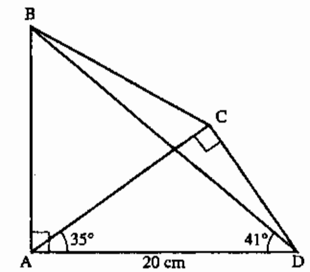


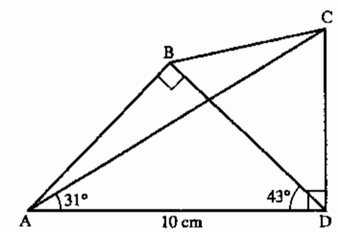
10.



11. $B\hat{A}D = A\hat{C}D = 90°$
$C\hat{A}D = 35°$
$B\hat{D}A = 41°$
AD = 20 cm
Calculate:
(a) AB
(b) DC
(c) BD



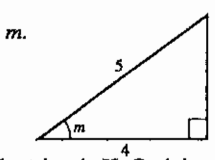
12. $A\hat{B}D = A\hat{D}C = 90°$
$C\hat{A}D = 31°$
$B\hat{D}A = 43°$
AD = 10 cm
Calculate:
(a) AB
(b) CD
(c) DB



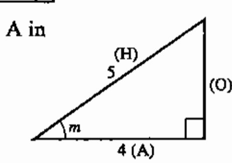
Finding an unknown angle

Example

Find the angle marked m.



(a) Label the sides of the triangle H, O, A in relation to angle m.



(b) In this example, we do not know 'O' so we need the cosine.

$$\cos m = \left(\frac{A}{H}\right) = \frac{4}{5}$$

(c) Change $\frac{4}{5}$ to a decimal: $\frac{4}{5} = 0{\cdot}8$

(d) $\cos m = 0{\cdot}8$
Find angle m from the cosine table: $m = 36{\cdot}9°$

Note: On a calculator, angles can be found as follows:

If $\cos m = \frac{4}{5}$

(a) Press [4] [÷] [5] [=]

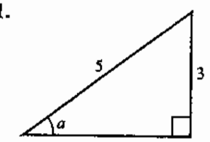
(b) Press [INV] and then [COS]

This will give the angle as 36·86989765°. We require the angle to 1 place of decimals so $m = 36{\cdot}9°$.

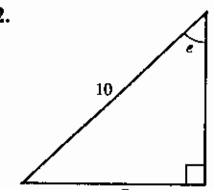
Exercise 4

In questions **1** to **15**, find the angle marked with a letter. All lengths are in cm.

1.



2.



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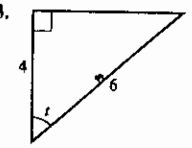

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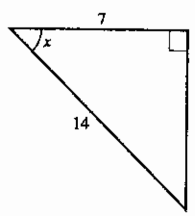
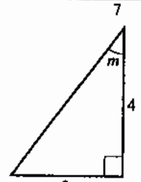


5.



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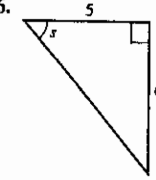

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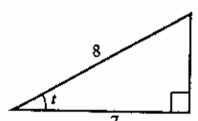

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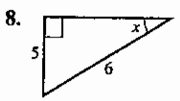

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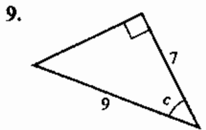

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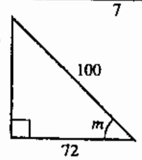

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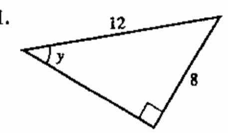

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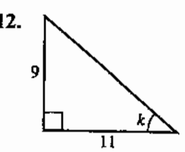

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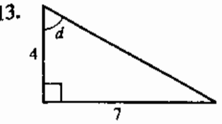

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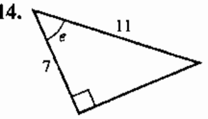

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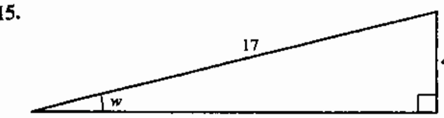
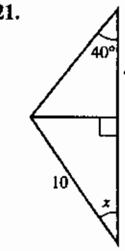


In questions **16** to **20**, the triangle has a right angle at the middle letter.

16. In △ABC, BC = 4, AC = 7. Find $\widehat{A}$.
17. In △DEF, EF = 5, DF = 10. Find $\widehat{F}$.
18. In △GHI, GH = 9, HI = 10. Find $\widehat{I}$.
19. In △JKL, JL = 5, KL = 3. Find $\widehat{J}$.
20. In △MNO, MN = 4, NO = 5. Find $\widehat{M}$.

In questions **21** to **26**, find the angle x.

21.



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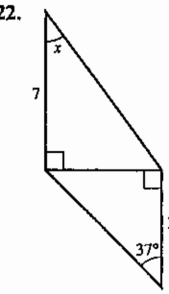

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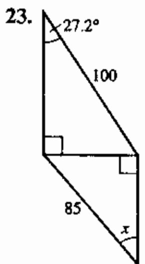

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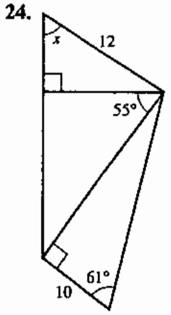

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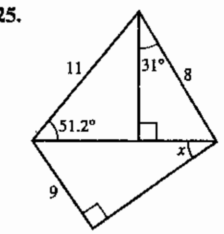

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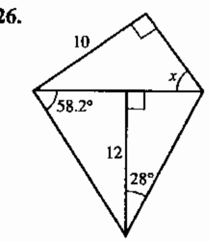
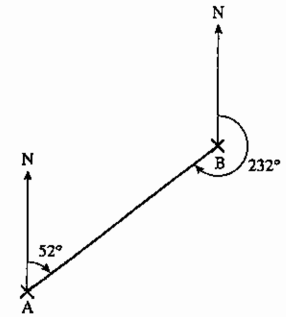


Bearings

A bearing is an angle measured clockwise from North. It is given using three digits.

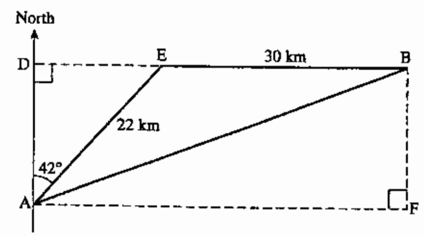
In the diagram:
the bearing of B from A is 052°
the bearing of A from B is 232°.



Example

A ship sails 22 km from A on a bearing of 042°, and a further 30 km on a bearing of 090° to arrive at B. What is the distance and bearing of B from A?

(a) Draw a clear diagram and label extra points as shown.



(b) Find DE and AD.

(i) $\sin 42° = \frac{DE}{22}$

$\therefore$ $DE = 22 \times \sin 42° = 14{\cdot}72$ km

(ii) $\cos 42° = \frac{AD}{22}$

$\therefore$ $AD = 22 \times \cos 42° = 16{\cdot}35$ km

(c) Using triangle ABF,
$AB^2 = AF^2 + BF^2$ (Pythagoras' Theorem)
and $AF = DE + EB$
$AF = 14{\cdot}72 + 30 = 44{\cdot}72$ km
and $BF = AD = 16{\cdot}35$ km

$\therefore$ $AB^2 = 44{\cdot}72^2 + 16{\cdot}35^2$
$= 2267{\cdot}2$

$AB = 47{\cdot}6$ km (to 3 S.F.)

(d) The bearing of B from A is given by the angle DAB.
But $D\hat{A}B = A\hat{B}F$.

$\tan A\hat{B}F = \frac{AF}{BF} = \frac{44{\cdot}72}{16{\cdot}35}$
$= 2{\cdot}7352$

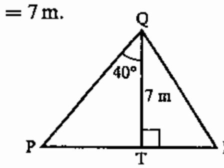
$\therefore$ $A\hat{B}F = 69{\cdot}9°$

B is 47·6 km from A on a bearing of 069·9°.

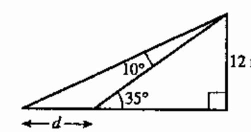
Exercise 5

In this exercise, start by drawing a clear diagram.

1. A ladder of length 6 m leans against a vertical wall so that the base of the ladder is 2 m from the wall. Calculate the angle between the ladder and the wall.
2. A ladder of length 8 m rests against a wall so that the angle between the ladder and the wall is 31°. How far is the base of the ladder from the wall?
3. A ship sails 35 km on a bearing of 042°.
   (a) How far north has it travelled?
   (b) How far east has it travelled?
4. A ship sails 200 km on a bearing of 243·7°.
   (a) How far south has it travelled?
   (b) How far west has it travelled?
5. Find TR if PR = 10 m and QT = 7 m.



6. Find *d*.



7. An aircraft flies 400 km from a point O on a bearing of 025° and then 700 km on a bearing of 080° to arrive at B.
   (a) How far north of O is B?
   (b) How far east of O is B?
   (c) Find the distance and bearing of B from O.
8. An aircraft flies 500 km on a bearing of 100° and then 600 km on a bearing of 160°.
   Find the distance and bearing of the finishing point from the starting point.

For questions **9** to **12**, plot the points for each question on a sketch graph with *x*- and *y*-axes drawn to the same scale.

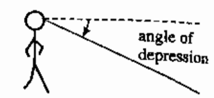
9. For the points A(5, 0) and B(7, 3), calculate the angle between AB and the x-axis.

10. For the points C(0, 2) and D(5, 9), calculate the angle between CD and the y-axis.

11. For the points A(3, 0), B(5, 2) and C(7, −2), calculate the angle BAC.

12. For the points P(2, 5), Q(5, 1) and R(0, −3), calculate the angle PQR.

13. From the top of a tower of height 75 m, a guard sees two prisoners, both due West of him. If the angles of depression of the two prisoners are 10° and 17°, calculate the distance between them.



14. An isosceles triangle has sides of length 8 cm, 8 cm and 5 cm. Find the angle between the two equal sides.

15. The angles of an isosceles triangle are 66°, 66° and 48°. If the shortest side of the triangle is 8·4 cm, find the length of one of the two equal sides.

16. A chord of length 12 cm subtends an angle of 78·2° at the centre of a circle. Find the radius of the circle.

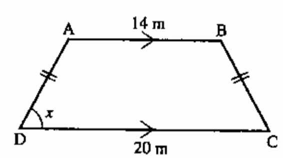
17. Find the acute angle between the diagonals of a rectangle whose sides are 5 cm and 7 cm.

18. A kite flying at a height of 55 m is attached to a string which makes an angle of 55° with the horizontal. What is the length of the string?

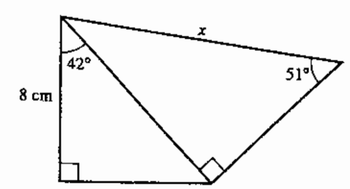
19. A boy is flying a kite from a string of length 150 m. If the string is taut and makes an angle of 67° with the horizontal, what is the height of the kite?

20. A rocket flies 10 km vertically, then 20 km at an angle of 15° to the vertical and finally 60 km at an angle of 26° to the vertical. Calculate the vertical height of the rocket at the end of the third stage.

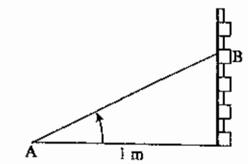
21. Find x, given $AD = BC = 6$ m.



22. Find x.



23. Ants can hear each other up to a range of 2 m. An ant at A, 1 m from a wall sees her friend at B about to be eaten by a spider. If the angle of elevation of B from A is 62°, will the spider have a meal or not? (Assume B escapes if he hears A calling.)



24. A hedgehog wishes to cross a road without being run over. He observes the angle of elevation of a lamp post on the other side of the road to be 27° from the edge of the road and 15° from a point 10 m back from the road. How wide is the road? If he can run at 1 m/s, how long will he take to cross?
If cars are travelling at 20 m/s, how far apart must they be if he is to survive?

25. From a point 10 m from a vertical wall, the angles of elevation of the bottom and the top of a statue of Sir Isaac Newton, set in the wall, are 40° and 52°. Calculate the height of the statue.

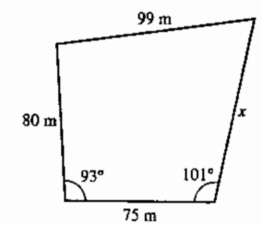
6.2 Scale drawing

On a scale drawing you must always state the scale you use.

Exercise 6

Make a scale drawing and then answer the questions.

1. A field has four sides as shown below:



How long is the side x in metres?

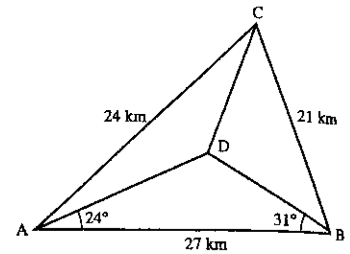
2. A destroyer and a cruiser leave a port at the same time. The destroyer sails at 38 knots on a bearing of 042° and the cruiser sails at 25 knots on a bearing of 315°. How far apart are the ships two hours later? [1 knot is a speed of 1 nautical mile per hour.]

3. Two radar stations A and B are 80 km apart and B is due East of A. One aircraft is on a bearing of 030° from A and 346° from B. A second aircraft is on a bearing of 325° from A and 293° from B. How far apart are the two aircraft?

4. A ship sails 95 km on a bearing of 140°, then a further 102 km on a bearing of 260° and then returns directly to its starting point. Find the length and bearing of the return journey.

5. A control tower observes the flight of an unidentified flying object.
At 09:23 the U.F.O. is 580 km away on a bearing of 043°.
At 09:25 the U.F.O. is 360 km away on a bearing of 016°.
What is the speed and the course of the U.F.O.?
[Use a scale of 1 cm to 50 km]

6. Make a scale drawing of the diagram below and find the length of CD in km.

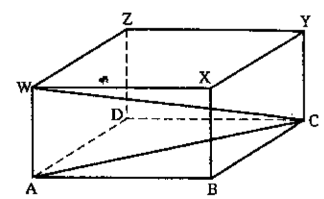


6.3 Three-dimensional problems

Always draw a large, clear diagram. It is often helpful to redraw the triangle which contains the length or angle to be found.

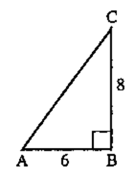
Example

A rectangular box with top WXYZ and base ABCD has AB = 6 cm, BC = 8 cm and WA = 3 cm.
Calculate:
(a) the length of AC
(b) the angle between WC and AC.



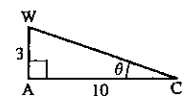
(a) Redraw triangle ABC.

$AC^2 = 6^2 + 8^2 = 100$
AC = 10 cm



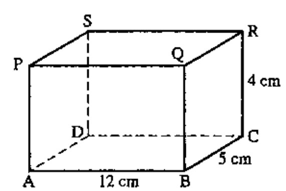
(b) Redraw triangle WAC.

Let $W\hat{C}A = \theta$
$\tan\theta = \frac{3}{10}$
$\theta = 16{\cdot}7°$
The angle between WC and AC is 16·7°.

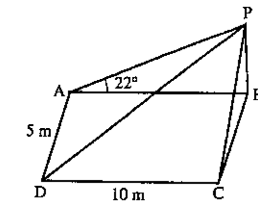


Exercise 7

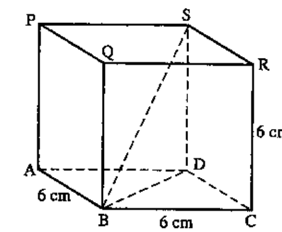
1. In the rectangular box shown, find:
(a) AC
(b) AR
(c) the angle between AC and AR.



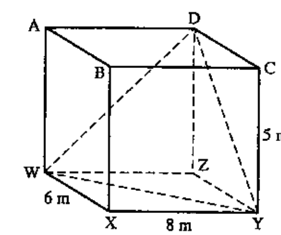
2. A vertical pole BP stands at one corner of a horizontal rectangular field as shown.
If AB = 10 m, AD = 5 m and the angle of elevation of P from A is 22°, calculate:
(a) the height of the pole
(b) the angle of elevation of P from C
(c) the length of a diagonal of the rectangle ABCD
(d) the angle of elevation of P from D.



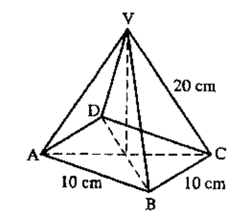
3. In the cube shown, find:
(a) BD
(b) AS
(c) BS
(d) the angle SBD
(e) the angle ASB



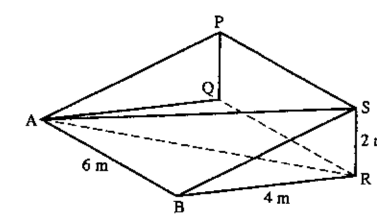
4. In the cuboid shown, find:
(a) WY
(b) DY
(c) WD
(d) the angle WDY



5. In the square-based pyramid, V is vertically above the middle of the base, AB = 10 cm and VC = 20 cm. Find:
(a) AC
(b) the height of the pyramid
(c) the angle between VC and the base ABCD
(d) the angle AVB
(e) the angle AVC

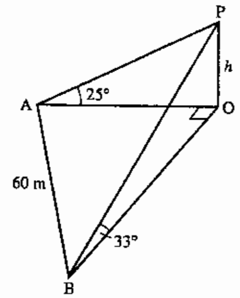


6. In the wedge shown, PQRS is perpendicular to ABRQ; PQRS and ABRQ are rectangles with AB = QR = 6 m, BR = 4 m, RS = 2 m. Find:
(a) BS
(b) AS
(c) angle BSR
(d) angle ASR
(e) angle PAS



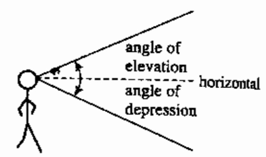
7. The edges of a box are 4 cm, 6 cm and 8 cm. Find the length of a diagonal and the angle it makes with the diagonal on the largest face.

8. In the diagram A, B and O are points in a horizontal plane and P is vertically above O, where OP = h m.



A is due West of O, B is due South of O and AB = 60 m. The angle of elevation of P from A is 25° and the angle of elevation of P from B is 33°.
(a) Find the length AO in terms of h.
(b) Find the length BO in terms of h.
(c) Find the value of h.

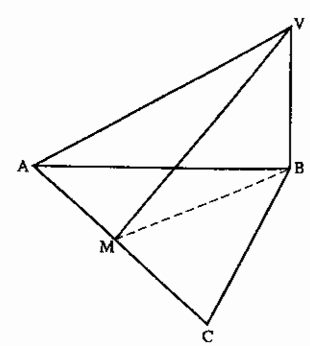
9. The angle of elevation of the top of a tower is 38° from a point A due South of it. The angle of elevation of the top of the tower from another point B, due East of the tower is 29°. Find the height of the tower if the distance AB is 50 m.



10. An observer at the top of a tower of height 15 m sees a man due West of him at an angle of depression 31°. He sees another man due South at an angle of depression 17°. Find the distance between the men.

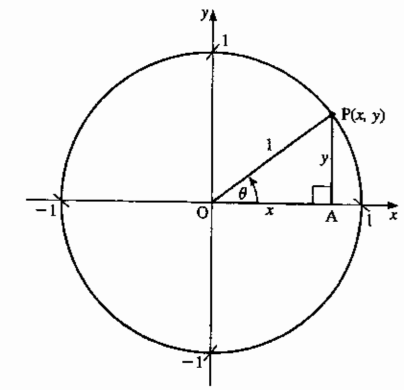
11. The angle of elevation of the top of a tower is 27° from a point A due East of it. The angle of elevation of the top of the tower is 11° from another point B due South of the tower. Find the height of the tower if the distance AB is 40 m.

12. The figure shows a triangular pyramid on a horizontal base ABC, V is vertically above B where VB = 10 cm, $A\hat{B}C = 90°$ and AB = BC = 15 cm. Point M is the mid-point of AC.
Calculate the size of angle VMB.



6.4 Sine, cosine, tangent for any angle

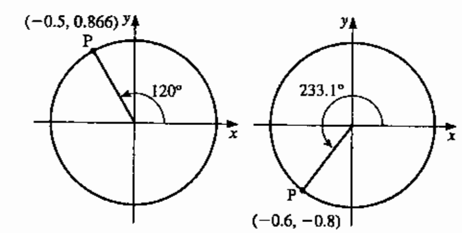
So far we have used sine, cosine and tangent only in right-angled triangles. For angles greater than 90°, we will see that there is a close connection between trigonometric ratios and circles.



The circle on the right is of radius 1 unit with centre (0, 0). A point P with coordinates (x, y) moves round the circumference of the circle. The angle that OP makes with the positive x-axis as it turns in an anticlockwise direction is θ.

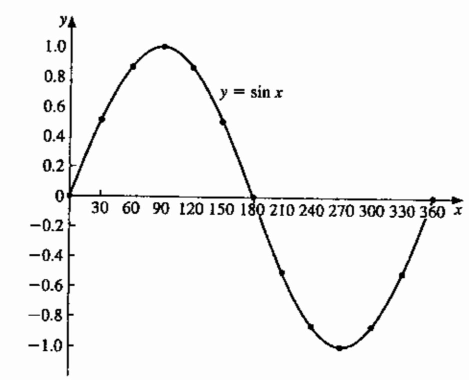
In triangle OAP, $\cos\theta = \dfrac{x}{1}$ and $\sin\theta = \dfrac{y}{1}$

The x-coordinate of P is $\cos\theta$.
The y-coordinate of P is $\sin\theta$.
This idea is used to define the cosine and the sine of any angle, including angles greater than 90°.
Here are two angles that are greater than 90°.



$\cos 120° = -0{\cdot}5$ $\qquad$ $\cos 233{\cdot}1° = -0{\cdot}6$
$\sin 120° = 0{\cdot}866$ $\qquad$ $\sin 233{\cdot}1° = -0{\cdot}8$

A graphics calculator can be used to show the graph of $y = \sin x$ for any range of angles. The graph on the right shows $y = \sin x$ for x from 0° to 360°. The curve above the x-axis has symmetry about $x = 90°$ and that below the x-axis has symmetry about $x = 270°$.



Note:
$\sin 150° = \sin 30°$ and $\cos 150° = -\cos 30°$
$\sin 110° = \sin 70°$ $\qquad$ $\cos 110° = -\cos 70°$
$\sin 163° = \sin 17°$ $\qquad$ $\cos 163° = -\cos 17°$

or $\sin x = \sin(180° - x)$
or $\cos x = -\cos(180° - x)$

These two results are particularly important for use with obtuse angles ($90° < x < 180°$) in Sections 6.5 and 6.6 when applying the sine formula or the cosine formula.

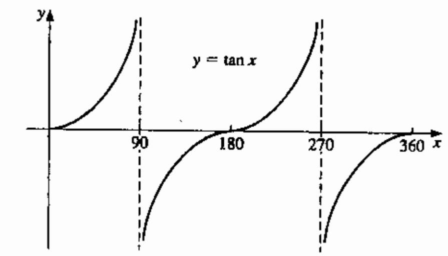
Exercise 8

1. (a) Use a calculator to find the cosine of all the angles 0°, 30°, 60°, 90°, 120°, ... 330°, 360°.
   (b) Draw a graph of $y = \cos x$ for $0 \leqslant x \leqslant 360°$. Use a scale of 1 cm to 30° on the x-axis and 5 cm to 1 unit on the y-axis.
2. Draw the graph of $y = \sin x$, using the same angles and scales as in Question 1.

In Questions 3 to 11 do not use a calculator. Use the symmetry of the graphs $y = \sin x$ and $y = \cos x$.
Angles are given to the nearest degree.

3. If sin 18° = 0·309, give another angle whose sine is 0·309.
4. If sin 27° = 0·454, give another angle whose sine is 0·454.
5. Give another angle which has the same sine as:
   (a) 40° (b) 70° (c) 130°
6. If cos 70° = 0·342, give another angle whose cosine is 0·342.
7. If cos 45° = 0·707, give another angle whose cosine is 0·707.
8. Give another angle which has the same cosine as:
   (a) 10° (b) 56° (c) 300°
9. If sin 20° = 0·342, what other angle has a sine of 0·342?
10. If sin 98° = 0·990, give another angle whose sine is 0·990.
11. If cos 120° = −0·5, give another angle whose cosine is −0·5.
12. Find two values for x, between 0° and 360°, if $\sin x = 0{\cdot}848$. Give each angle to the nearest degree.
13. If $\sin x = 0{\cdot}35$, find two solutions for x between 0° and 360°.
14. If $\cos x = 0{\cdot}6$, find two solutions for x between 0° and 360°.
15. Find two solutions between 0° and 360°:
   (a) $\sin x = 0{\cdot}72$ (b) $\cos x = 0{\cdot}3$
   (c) $\cos x = 0{\cdot}7$ (d) $\sin x = -0{\cdot}65$
16. Find *four* solutions of the equation
   $(\sin x)^2 = \frac{1}{4}$ for x between 0° and 360°.
17. Draw the graph of $y = 2\sin x + 1$ for $0 \leqslant x \leqslant 180°$, taking 1 cm to 10° for x and 5 cm to 1 unit for y. Find approximate solutions to the equations:
   (a) $2\sin x + 1 = 2{\cdot}3$
   (b) $\dfrac{1}{(2\sin x + 1)} = 0{\cdot}5$

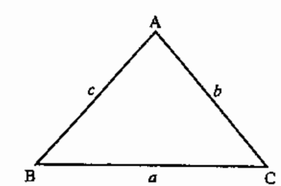
18. Draw the graph of $y = 2\sin x + \cos x$ for $0 \leqslant x \leqslant 180°$, taking 1 cm to 10° for x and 5 cm to 1 unit for y.
   (a) Solve approximately the equations:
      (i) $2\sin x + \cos x = 1{\cdot}5$
      (ii) $2\sin x + \cos x = 0$
   (b) Estimate the maximum value of y.
   (c) Find the value of x at which the maximum occurs.
19. Draw the graph of $y = 3\cos x - 4\sin x$ for $0° \leqslant x \leqslant 220°$, taking 1 cm to 10° for x and 2 cm to 1 unit for y.
   Solve approximately the equations:
   (a) $3\cos x - 4\sin x + 1 = 0$
   (b) $3\cos x = 4\sin x$
20. Find the tangents of the angles 0°, 20°, 40°, 60°, ... 320°, 340°, 360°.
   (a) Notice that we have deliberately omitted 90° and 270°. Why has this been done?
   (b) Draw a graph of $y = \tan x$. Use a scale of 1 cm to 20° on the x-axis and 1 cm to 1 unit on the y-axis.
   (c) Draw a vertical dotted line at $x = 90°$ and $x = 270°$. These lines are *asymptotes* to the curve. As the value of x approaches 90° from either side, the curve gets nearer and nearer to the asymptote but it *never* quite reaches it.



6.5 The sine rule

The sine rule enables us to calculate sides and angles in some triangles where there is not a right angle.

In △ABC, we use the convention that
a is the side opposite $\hat{A}$
b is the side opposite $\hat{B}$, etc.



Sine rule

either $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ ...[1]

or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ ...[2]

Use [1] when finding a *side*,
and [2] when finding an *angle*.

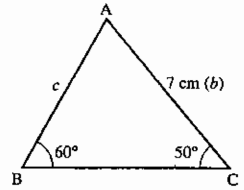
Example 1

Find c.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 50°} = \frac{7}{\sin 60°}$$

$$c = \frac{7 \times \sin 50°}{\sin 60°} = 6{\cdot}19\text{ cm} \quad (3\text{ S.F.})$$



Although we cannot have an angle of more than 90° in a right-angled triangle, it is still useful to define sine, cosine and tangent for these angles.
For an obtuse angle x,
we have $\sin x = \sin(180 - x)$

Examples: $\sin 130° = \sin 50°$
$\sin 170° = \sin 10°$
$\sin 116° = \sin 64°$

Most people simply use a calculator when finding the sine of an obtuse angle.

Example 2

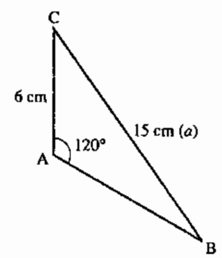
Find $\hat{B}$.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{6} = \frac{\sin 120°}{15} \quad (\sin 120° = \sin 60°)$$

$$\sin B = \frac{6 \times \sin 60°}{15}$$

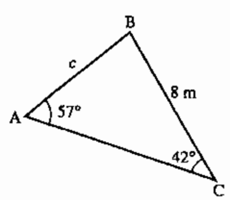
$$\sin B = 0{\cdot}346$$
$$\hat{B} = 20{\cdot}3°$$



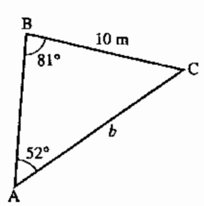
Exercise 9

For questions **1** to **6**, find each side marked with a letter.
Give answers to 3 S.F.

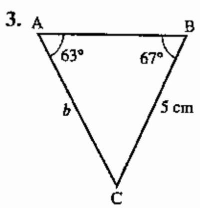
1.



2.



3.



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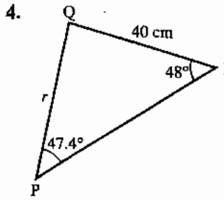

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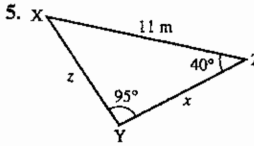

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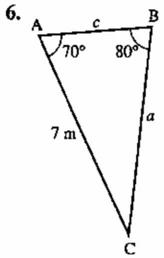


7. In $\triangle ABC$, $\hat{A} = 61°$, $\hat{B} = 47°$, $AC = 7{\cdot}2$ cm. Find BC.

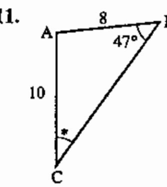
8. In $\triangle XYZ$, $\hat{Z} = 32°$, $\hat{Y} = 78°$, $XY = 5{\cdot}4$ cm. Find XZ.

9. In $\triangle PQR$, $\hat{Q} = 100°$, $\hat{R} = 21°$, $PQ = 3{\cdot}1$ cm. Find PR.

10. In $\triangle LMN$, $\hat{L} = 21°$, $\hat{N} = 30°$, $MN = 7$ cm. Find LN.

In questions 11 to 18, find each angle marked *. All lengths are in centimetres.

11.



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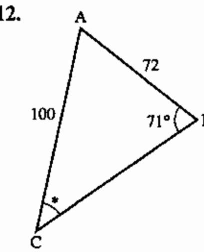

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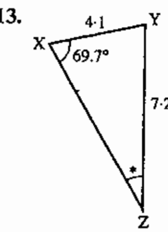

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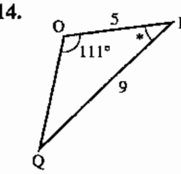

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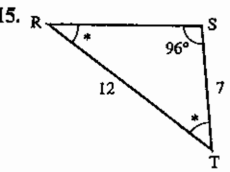

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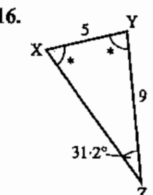
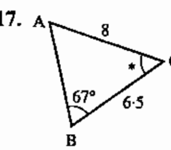


17.



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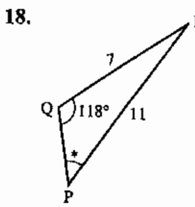


19. In $\triangle ABC$, $\hat{A} = 62°$, $BC = 8$, $AB = 7$.
Find $\hat{C}$.

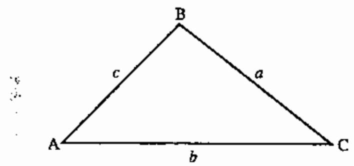
20. In $\triangle XYZ$, $\hat{Y} = 97{\cdot}3°$, $XZ = 22$, $XY = 14$.
Find $\hat{Z}$.

21. In $\triangle DEF$, $\hat{D} = 58°$, $EF = 7{\cdot}2$, $DE = 5{\cdot}4$.
Find $\hat{F}$.

22. In $\triangle LMN$, $\hat{M} = 127{\cdot}1°$, $LN = 11{\cdot}2$, $LM = 7{\cdot}3$. Find $\hat{L}$.

6.6 The cosine rule

We use the cosine rule when we have either
(a) two sides and the included angle or
(b) all three sides.



There are two forms.

1. To find the length of a side.

$a^2 = b^2 + c^2 - (2bc \cos A)$
or $b^2 = c^2 + a^2 - (2ac \cos B)$
or $c^2 = a^2 + b^2 - (2ab \cos C)$

2. To find an angle when given all three sides.

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
or $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

For an obtuse angle x we have $\cos x = -\cos(180 - x)$

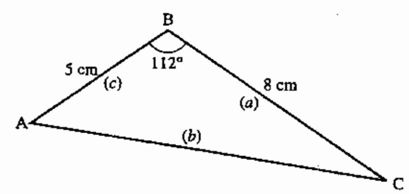
Examples $\cos 120° = -\cos 60°$
$\cos 142° = -\cos 38°$

Example 1

Find b.
$b^2 = a^2 + c^2 - (2ac \cos B)$
$b^2 = 8^2 + 5^2 - (2 \times 8 \times 5 \times \cos 112°)$
$b^2 = 64 + 25 - [80 \times (-0{\cdot}3746)]$
$b^2 = 64 + 25 + 29{\cdot}968$

(Notice the change of sign for the obtuse angle)

$b = \sqrt{(118{\cdot}968)} = 10{\cdot}9$ cm (to 3 s.f.)



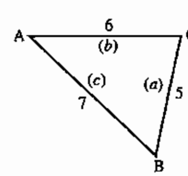
Example 2

Find angle C.

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

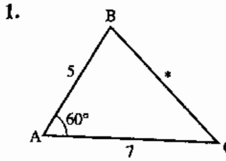
$\cos C = \dfrac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6} = \dfrac{12}{60} = 0{\cdot}200$

$\hat{C} = 78{\cdot}5°$



Exercise 10

Find the sides marked *. All lengths are in centimetres.

1.


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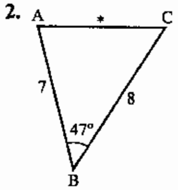

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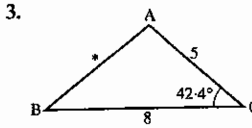

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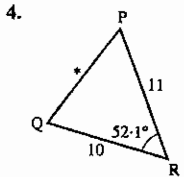

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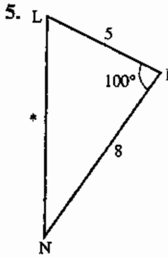

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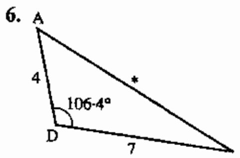


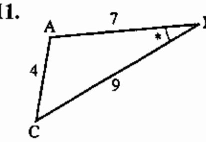
7. In △ABC, AB = 4 cm, AC = 7 cm, $\hat{A} = 57°$. Find BC.

8. In △XYZ, XY = 3 cm, YZ = 3 cm, $\hat{Y} = 90°$. Find XZ.

9. In △LMN, LM = 5·3 cm, MN = 7·9 cm, $\hat{M} = 127°$. Find LN.

10. △PQR, $\hat{Q} = 117°$, PQ = 80 cm, QR = 100 cm. Find PR.

In questions **11** to **16**, find each angle marked *.

11.


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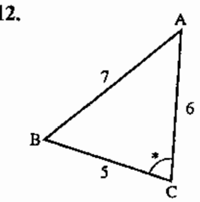

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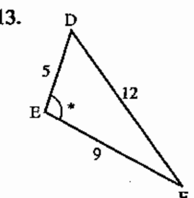

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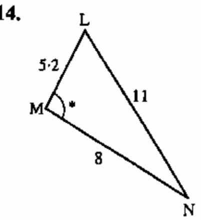

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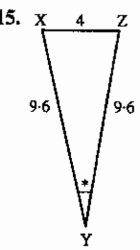

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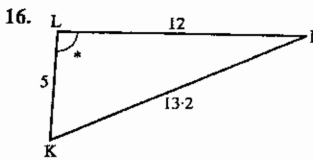


17. In $\triangle ABC$, $a = 4{\cdot}3$, $b = 7{\cdot}2$, $c = 9$. Find $\widehat{C}$.

18. In $\triangle DEF$, $d = 30$, $e = 50$, $f = 70$. Find $\widehat{E}$.

19. In $\triangle PQR$, $p = 8$, $q = 14$, $r = 7$. Find $\widehat{Q}$.

20. In $\triangle LMN$, $l = 7$, $m = 5$, $n = 4$. Find $\widehat{N}$.

21. In $\triangle XYZ$, $x = 5{\cdot}3$, $y = 6{\cdot}7$, $z = 6{\cdot}14$. Find $\widehat{Z}$.

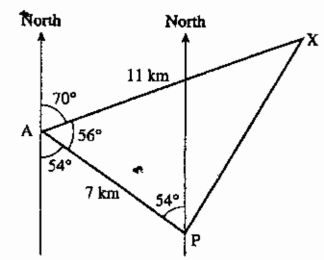
22. In $\triangle ABC$, $a = 4{\cdot}1$, $c = 6{\cdot}3$, $\widehat{B} = 112{\cdot}2°$. Find b.

23. In $\triangle PQR$, $r = 0{\cdot}72$, $p = 1{\cdot}14$, $\widehat{Q} = 94{\cdot}6°$. Find q.

24. In $\triangle LMN$, $n = 7{\cdot}206$, $l = 6{\cdot}3$, $\widehat{L} = 51{\cdot}2°$, $\widehat{N} = 63°$. Find m.

Example

A ship sails from a port P a distance of 7 km on a bearing of 306° and then a further 11 km on a bearing of 070° to arrive at X. Calculate the distance from P to X.



$$PX^2 = 7^2 + 11^2 - (2 \times 7 \times 11 \times \cos 56°)$$
$$= 49 + 121 - (86{\cdot}12)$$

$$PX^2 = 83{\cdot}88$$
$$PX = 9{\cdot}16 \text{ km (to 3 s.f.)}$$

The distance from P to X is 9·16 km.

Exercise 11

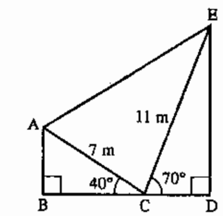
Start each question by drawing a large, clear diagram.

1. In triangle PQR, $\widehat{Q} = 72°$, $\widehat{R} = 32°$ and PR = 12 cm. Find PQ.
2. In triangle LMN, $\widehat{M} = 84°$, LM = 7 m and MN = 9 m. Find LN.
3. A destroyer D and a cruiser C leave port P at the same time. The destroyer sails 25 km on a bearing 040° and the cruiser sails 30 km on a bearing of 320°. How far apart are the ships?
4. Two honeybees A and B leave the hive H at the same time; A flies 27 m due South and B flies 9 m on a bearing of 111°. How far apart are they?
5. Find all the angles of a triangle in which the sides are in the ratio 5 : 6 : 8.
6. A golfer hits his ball B a distance of 170 m towards a hole H which measures 195 m from the tee T to the green. If his shot is directed 10° away from the true line to the hole, find the distance between his ball and the hole.

7. From A, B lies 11 km away on a bearing of 041° and C lies 8 km away on a bearing of 341°. Find:
   (a) the distance between B and C
   (b) the bearing of B from C.

8. From a lighthouse L an aircraft carrier A is 15 km away on a bearing of 112° and a submarine S is 26 km away on a bearing of 200°. Find:
   (a) the distance between A and S
   (b) the bearing of A from S.

9. If the line BCD is horizontal find:
   (a) AE
   (b) $E\widehat{A}C$

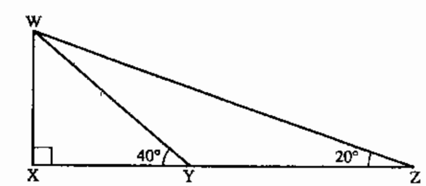


   (c) the angle of elevation of E from A.

10. An aircraft flies from its base 200 km on a bearing 162°, then 350 km on a bearing 260°, and then returns directly to base. Calculate the length and bearing of the return journey.

11. Town Y is 9 km due North of town Z. Town X is 8 km from Y, 5 km from Z and somewhere to the west of the line YZ.
    (a) Draw triangle XYZ and find angle YZX.
    (b) During an earthquake, town X moves due South until it is due West of Z. Find how far it has moved.

12. Calculate WX, given YZ = 15 m.

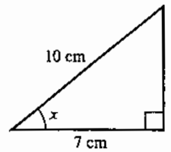


13. A golfer hits her ball a distance of 127 m so that it finishes 31 m from the hole. If the length of the hole is 150 m, calculate the angle between the line of her shot and the direct line to the hole.

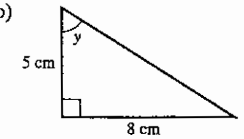
Revision exercise 6A

1. Calculate the side or angle marked with a letter.

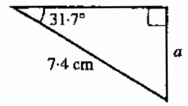
(a)



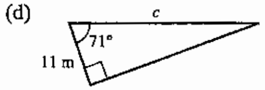
(b)



(c)



(d)



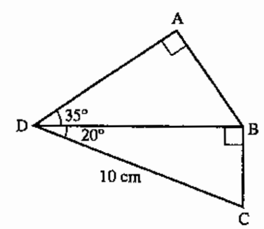
2. Given that x is an acute angle and that

$$3\tan x - 2 = 4\cos 35{\cdot}3°$$

calculate:
(a) $\tan x$
(b) the value of x in degrees correct to 1 D.P.

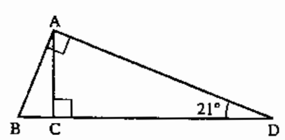
3. In the triangle XYZ, XY = 14 cm, XZ = 17 cm and angle YXZ = 25°. A is the foot of the perpendicular from Y to XZ. Calculate:
(a) the length XA
(b) the length YA
(c) the angle ZYA

4. Calculate the length of AB.



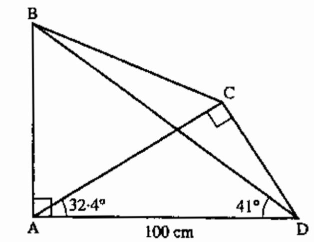
5. (a) A lies on a bearing of 040° from B. Calculate the bearing of B from A.
(b) The bearing of X from Y is 115°. Calculate the bearing of Y from X.

6. Given BD = 1 m, calculate the length AC.



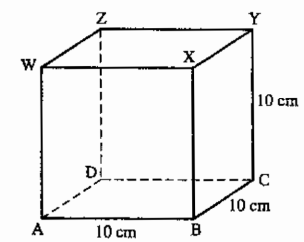
7. In the triangle PQR, angle PQR = 90° and angle RPQ = 31°. The length of PQ is 11 cm. Calculate:
(a) the length of QR
(b) the length of PR
(c) the length of the perpendicular from Q to PR.

8. $B\hat{A}D = D\hat{C}A = 90°$, $C\hat{A}D = 32{\cdot}4°$, $B\hat{D}A = 41°$ and AD = 100 cm.
Calculate:
(a) the length of AB
(b) the length of DC
(c) the length of BD.

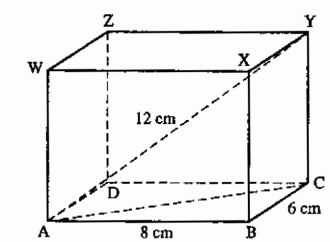


9. An observer at the top of a tower of height 20 m sees a man due East of him at an angle of depression of 27°. He sees another man due South of him at an angle of depression of 30°. Find the distance between the men on the ground.

10. The figure shows a cube of side 10 cm.
Calculate:
(a) the length of AC
(b) the angle YAC
(c) the angle ZBD.



11. The diagram shows a rectangular block.
AY = 12 cm, AB = 8 cm, BC = 6 cm.
Calculate:
(a) the length YC
(b) the angle $Y\hat{A}Z$



12. VABCD is a pyramid in which the base ABCD is a square of side 8 cm; V is vertically above the centre of the square and VA = VB = VC = VD = 10 cm.
Calculate:
(a) the length AC
(b) the height of V above the base
(c) the angle $V\hat{C}A$.

Questions **13** to **18** may be answered either by scale drawing or by using the sine and cosine rules.

13. Two lighthouses A and B are 25 km apart and A is due West of B. A submarine S is on a bearing of 137° from A and on a bearing of 170° from B. Find the distance of S from A and the distance of S from B.

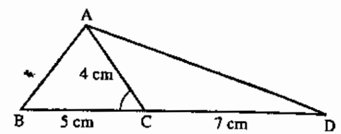
14. In triangle PQR, PQ = 7 cm, PR = 8 cm and QR = 9 cm. Find angle QPR.

15. In triangle XYZ, XY = 8 m, $\hat{X} = 57°$ and $\hat{Z} = 50°$. Find the lengths YZ and XZ.

16. In triangle ABC, $\hat{A} = 22°$ and $\hat{C} = 44°$.

Find the ratio $\frac{BC}{AB}$.

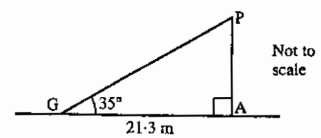
17. Given cos $A\hat{C}B = 0{\cdot}6$, AC = 4 cm, BC = 5 cm and CD = 7 cm, find the length of AB and AD.



18. Find the smallest angle in a triangle whose sides are of length $3x$, $4x$ and $6x$.

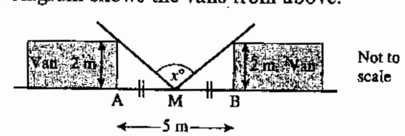
Examination exercise 6B

1.



A wire, GP, connects the top of a vertical pole, AP, to the horizontal ground.
GA = 21·3 m and angle PGA = 35°.
Calculate GP, the length of the wire. J 97 2

2. Two vans, 5 m apart and each 2 m wide, are parked at the side of a road. The diagram shows the vans from above.

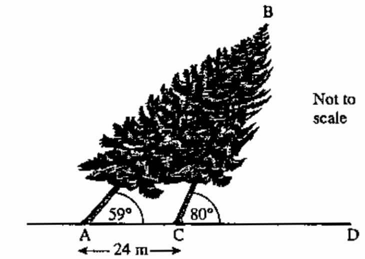


(a) A man stands on the pavement at M, halfway between A and B. Calculate his angle of view ($x°$).
(b) Calculate his angle of view if he stood at the point B. N 98 2

3. $\cos A = \sqrt{\dfrac{1}{4 - 2\sqrt{2}}}$

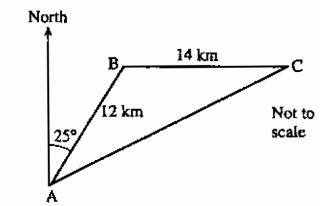
Calculate the value of angle A. N 98 2

4.



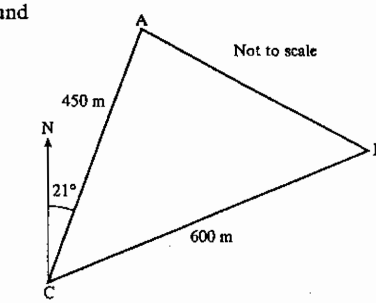
During a storm, a tree, AB, is blown over and rests on another tree CB. $B\hat{A}C = 59°$, $B\hat{C}D = 80°$, AC = 24 m and ACD is horizontal.
Calculate the length AB. J 95 2

5.



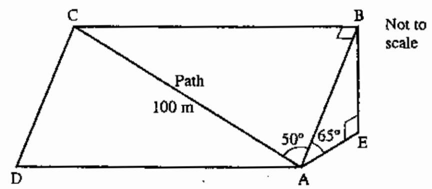
Hussein travels 12 km from A to B on a bearing of 025°.
He then travels due East for 14 km to C.
(a) Show that angle ABC is 115°.
(b) Calculate:
 (i) the distance AC,
 (ii) the angle BAC,
 (iii) the bearing of A from C. J 97 4

6. The diagram represents three straight roads which surround a village.
The bearing of A from C is 021°. Angle ACB = 41°.
The lengths of the roads CA and CB are 450 m and 600 m respectively.
(a) Calculate the bearing of
 (i) B from C,
 (ii) C from A.
(b) Calculate how far A is north of C.
(c) Calculate the length of the road AB.
(d) The area ABC contains homes for 374 people.
 Calculate the average number of people per hectare in the area. (1 hectare = 10 000 m^2.)



N 95 4

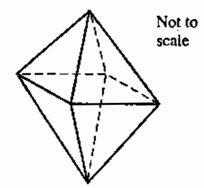
7.



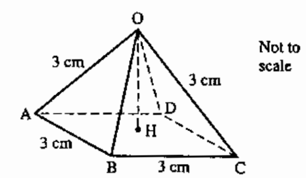
On a hillside ABCD a path, AC, is 100 m long.
The path makes an angle of 50° with AB. Angle ABC = 90°.
(a) Calculate the length of AB.
(b) The hillside slopes upwards at an angle of 65° and angle AEB = 90°.
Calculate the height, BE, of the hill.

N 97 2

8.



The diagram shows a regular octahedron. All the edges are 3 cm long.
(a) For this solid, write down the number of:
 (i) faces,
 (ii) vertices,
 (iii) edges.
(b) The octahedron is split into two equal parts.
One of the parts is shown in the diagram on the right.
Calculate:
 (i) the length of AC,
 (ii) the vertical height OH,
 (iii) the angle between OA and the base ABCD.
(c) The volume of a pyramid is $\frac{1}{3}$ base area × height.
Calculate the volume of the octahedron.



N 98 4

9. Find x when $\sin x° = -0{\cdot}866$, $\cos x° = -0{\cdot}5$ and $0 \leqslant x \leqslant 360$.

J 98 2

7 GRAPHS

Rene Descartes (1596–1650) was one of the greatest philosophers of his time. Strangely his restless mind only found peace and quiet as a soldier and he apparently discovered the idea of 'cartesian' geometry in a dream before the battle of Prague. The word 'cartesian' is derived from his name and his work formed the link between geometry and algebra which inevitably led to the discovery of calculus. He finally settled in Holland for ten years, but later moved to Sweden where he soon died of pneumonia.

7.1 Drawing accurate graphs

Example

Draw the graph of $y = 2x - 3$ for values of x from -2 to $+4$.

(a) The coordinates of points on the line are calculated in a table.

x	−2	−1	0	1	2	3	4
$2x$	−4	−2	0	2	4	6	8
−3	−3	−3	−3	−3	−3	−3	−3
y	−7	−5	−3	−1	1	3	5

(b) Draw and label axes using suitable scales.
(c) Plot the points and draw a pencil line through them. Label the line with its equation.