UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER

PROGRAMME: B.SC. IN AGRICULTURAL ECONOMICS AND AGRIBUSINESS MANAGEMENT III (NEW PROGRAMME)

COURSE CODE: AEM 308

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

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QUESTION 1

(a) The deviations of the observations from the regression line may be attributed to several factors. *List* and *briefly discuss five* (5) of these factors. [10 marks]

(b) Draw a *schematic diagram* to show the procedure to be followed when *testing* an *economic theory*. [16 marks]

(c) State with reason whether the following statements are true, false, or uncertain. Be precise. (2 marks each) [14 marks total]

(i) The *t* test of significance for the regression coefficients of a three-variable regression model requires that the sampling distributions of estimators of β_0 and β_1 follow the normal distribution.

(ii) Even though the disturbance term in the Classical Linear Regression Model (CLRM) is not normally distributed, the OLS estimators are still unbiased.

(iii) If there is no intercept in the regression model, the estimated u_i will not sum to zero.

(iv) The *p* value and the size of a test statistic mean the same thing.

(v) In a regression model that contains the intercept, the sum of the residuals is always zero.

(vi) If a null hypothesis is not rejected, it is true.

(vii) The higher is the value of σ^2, the larger is the variance of the estimator of β_1.

QUESTION 2

The following table includes the gross national product (X) and the demand for food (Y) measured in arbitrary units, in an underdeveloped country over the 10-year period 1960-1969.

Year	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**	**1970**
Y	6	7	8	10	8	9	10	9	11	10
X	50	52	55	59	57	58	62	65	68	70

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 98

Intermediate results

$\sum X = 596$ $\sum Y = 88$ $\sum X^2 = 35{,}916$ $\sum Y^2 = 796$

$\sum XY = 5{,}325$

(a) *Estimate* the *food function.* [13 marks]

(b) Compute the *standard error* of the *estimate* of the *regression coefficient* and conduct a test of significance at the 5 per cent level of significance. If appropriate, interpret the results. [10 marks]

(c) Find the 99 per cent confidence interval for the population (true) regression coefficient. [7 marks]

QUESTION 3

The following table shows the values of expenditure on clothing (Y), total expenditure (X_1) and the price of clothing (X_2).

Year	Expenditure on clothing Y	Total expenditure X_1	Price of clothing X_2
1960	3.5	15	16
1961	4.3	20	13
1962	5	30	10
1963	6	42	7
1964	7	50	7
1965	9	54	5
1966	8	65	4
1967	10	72	3
1968	12	85	3.5
1969	14	90	2

(a) Fit a non-linear function of the constant elasticity type

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} e^U$$

[20 marks]

(b) Conduct a test of the overall significance of the regression at the 5% level of significance. Provide an economic interpretation of the results of your test.

[10 marks]

QUESTION 4

In studying the farm demand for tractors, Griliches used the following model:

$$T_t^* = \alpha (X_{1,t-1})^{\beta 1} (X_{2,t-1})^{\beta 2} \quad (1)$$

where
T^* = desired stock of tractors
X_1= relative price of tractors
X_2 = interest rate

Taking log of both sides of (1) gives

$$\log T_t^* = \log\alpha + \beta_1 \log X_{1,t-1} + \beta_2 \log X_{2,t-1} \quad (2)$$

Since the desired stock of tractors is not directly observable, Griliches used the following stock adjustment model:

$$\frac{T_t}{T_{t-1}} = \left[\frac{T_t^*}{T_{t-1}} \right]^{\delta} \qquad 0 < \delta \leq 1 \quad (3)$$

By taking the log of both sides of (3), the stock adjustment model can be written as follows:

$$\log T_t - \log T_{t-1} = \delta (\log T_t^* - \log T_{t-1}) \quad (4)$$

Griliches obtained the following results for the period 1921 - 1957:

$$\text{Estimated } \log T_t = \text{const} + \underset{(0.051)}{0.218} \log X_{1,t-1} - \underset{(0.170)}{0.855} \log X_{2,t-1} + \underset{(0.035)}{0.864} \log T_{t-1}$$

$R^2 = 0.987$

where the figures in the parentheses are the estimated standard errors.

a. What is the estimated coefficient of adjustment? [5 marks]
b. What are the short- and long-run price elasticities? [10 marks]
c. What are the short- and long-run interest elasticities? [10 marks]
d. What are the reasons for high or low rate of adjustment in the present model? [5 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X \sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1 \overline{X}$$

$$r^2 = \hat{\beta}_1^2 \frac{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y \sum Y\right)}, \qquad F = \frac{r^2}{1 - r^2}(n-2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u^2 \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\sigma_u^2 \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2}\dfrac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n > 30$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2}\dfrac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n > 30$

$$t = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2}\dfrac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n \leq 30$

$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2}\dfrac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n \leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

$$\text{Total SS} = \sum Y^2 - n\overline{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\overline{Y}^2,$$

$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n - k - 1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n - k - 1} = \frac{\text{Total SS} - \text{Regression SS}}{n - k - 1},$$

$$\hat{\sigma}_{(\hat{\beta}_j)} = \sqrt{(\text{j}+1)\text{th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0, 1, \ldots, k.$$

Appendix E Points for the Distribution of F [5% (light type) and 1% (bold face type)]

f_2	f_1 Degrees of freedom (for greater mean square)																								f_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
1	161	200	216	225	230	234	237	239	241	242	243	244	245	246	248	249	250	251	252	253	253	254	254	254	1
	4,052	**4,999**	**5,403**	**5,625**	**5,764**	**5,859**	**5,928**	**5,981**	**6,022**	**6,056**	**6,082**	**6,106**	**6,142**	**6,169**	**6,208**	**6,234**	**6,261**	**6,286**	**6,302**	**6,323**	**6,334**	**6,352**	**6,361**	**6,366**	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41	19.42	19.43	19.44	19.45	19.46	19.47	19.47	19.48	19.49	19.49	19.50	19.50	2
	98.49	**99.00**	**99.17**	**99.25**	**99.30**	**99.33**	**99.36**	**99.37**	**99.39**	**99.40**	**99.41**	**99.42**	**99.43**	**99.44**	**99.45**	**99.46**	**99.47**	**99.48**	**99.48**	**99.49**	**99.49**	**99.49**	**99.50**	**99.50**	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74	8.71	8.69	8.66	8.64	8.62	8.60	8.58	8.57	8.56	8.54	8.54	8.53	3
	34.12	**30.82**	**29.46**	**28.71**	**28.24**	**27.91**	**27.67**	**27.49**	**27.34**	**27.23**	**27.13**	**27.05**	**26.92**	**26.83**	**26.69**	**26.60**	**26.50**	**26.41**	**26.35**	**26.27**	**26.23**	**26.18**	**26.14**	**26.12**	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91	5.87	5.84	5.80	5.77	5.74	5.71	5.70	5.68	5.66	5.65	5.64	5.63	4
	21.20	**18.00**	**16.69**	**15.98**	**15.52**	**15.21**	**14.98**	**14.80**	**14.66**	**14.54**	**14.45**	**14.37**	**14.24**	**14.15**	**14.02**	**13.93**	**13.83**	**13.74**	**13.69**	**13.61**	**13.57**	**13.52**	**13.48**	**13.46**	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.70	4.68	4.64	4.60	4.56	4.53	4.50	4.46	4.44	4.42	4.40	4.38	4.37	4.36	5
	16.26	**13.27**	**12.06**	**11.39**	**10.97**	**10.67**	**10.45**	**10.29**	**10.15**	**10.05**	**9.96**	**9.89**	**9.77**	**9.68**	**9.55**	**9.47**	**9.38**	**9.29**	**9.24**	**9.17**	**9.13**	**9.07**	**9.04**	**9.02**	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.96	3.92	3.87	3.84	3.81	3.77	3.75	3.72	3.71	3.69	3.68	3.67	6
	13.74	**10.92**	**9.78**	**9.15**	**8.75**	**8.47**	**8.26**	**8.10**	**7.98**	**7.87**	**7.79**	**7.72**	**7.60**	**7.52**	**7.39**	**7.31**	**7.23**	**7.14**	**7.09**	**7.02**	**6.99**	**6.94**	**6.90**	**6.88**	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57	3.52	3.49	3.44	3.41	3.38	3.34	3.32	3.29	3.28	3.25	3.24	3.23	7
	12.25	**9.55**	**8.45**	**7.85**	**7.46**	**7.19**	**7.00**	**6.84**	**6.71**	**6.62**	**6.54**	**6.47**	**6.35**	**6.27**	**6.15**	**6.07**	**5.98**	**5.90**	**5.85**	**5.78**	**5.75**	**5.70**	**5.67**	**5.65**	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28	3.23	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96	2.94	2.93	8
	11.26	**8.65**	**7.59**	**7.01**	**6.63**	**6.37**	**6.19**	**6.03**	**5.91**	**5.82**	**5.74**	**5.67**	**5.56**	**5.48**	**5.36**	**5.28**	**5.20**	**5.11**	**5.06**	**5.00**	**4.96**	**4.91**	**4.88**	**4.86**	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07	3.02	2.98	2.93	2.90	2.86	2.82	2.80	2.77	2.76	2.73	2.72	2.71	9
	10.56	**8.02**	**6.99**	**6.42**	**6.06**	**5.80**	**5.62**	**5.47**	**5.35**	**5.26**	**5.18**	**5.11**	**5.00**	**4.92**	**4.80**	**4.73**	**4.64**	**4.56**	**4.51**	**4.45**	**4.41**	**4.36**	**4.33**	**4.31**	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91	2.86	2.82	2.77	2.74	2.70	2.67	2.64	2.61	2.59	2.56	2.55	2.54	10
	10.04	**7.56**	**6.55**	**5.99**	**5.64**	**5.39**	**5.21**	**5.06**	**4.95**	**4.85**	**4.78**	**4.71**	**4.60**	**4.52**	**4.41**	**4.33**	**4.25**	**4.17**	**4.12**	**4.05**	**4.01**	**3.96**	**3.93**	**3.91**	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79	2.74	2.70	2.65	2.61	2.57	2.53	2.50	2.47	2.45	2.42	2.41	2.40	11
	9.65	**7.20**	**6.22**	**5.67**	**5.32**	**5.07**	**4.88**	**4.74**	**4.63**	**4.54**	**4.46**	**4.40**	**4.29**	**4.21**	**4.10**	**4.02**	**3.94**	**3.86**	**3.80**	**3.74**	**3.70**	**3.66**	**3.62**	**3.60**	
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	2.72	2.69	2.64	2.60	2.54	2.50	2.46	2.42	2.40	2.36	2.35	2.32	2.31	2.30	12
	9.33	**6.93**	**5.95**	**5.41**	**5.06**	**4.82**	**4.65**	**4.50**	**4.39**	**4.30**	**4.22**	**4.16**	**4.05**	**3.98**	**3.86**	**3.78**	**3.70**	**3.61**	**3.56**	**3.49**	**3.46**	**3.41**	**3.38**	**3.36**	
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	2.63	2.60	2.55	2.51	2.46	2.42	2.38	2.34	2.32	2.28	2.26	2.24	2.22	2.21	13
	9.07	**6.70**	**5.74**	**5.20**	**4.86**	**4.62**	**4.44**	**4.30**	**4.19**	**4.10**	**4.02**	**3.96**	**3.85**	**3.78**	**3.67**	**3.59**	**3.51**	**3.42**	**3.37**	**3.30**	**3.27**	**3.21**	**3.16**	**3.16**	

continued next page

Appendix C Distribution of *t* Probability

n	.9	.8	.7	.6	.5	.4	.3	.2	.1	.05	.02	.01	.001
1	.158	.325	.510	.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	.142	.289	.445	.617	.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	.137	.277	.424	.584	.765	.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	.134	.271	.414	.569	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	.132	.267	.408	.559	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	.131	.265	.404	.553	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	.130	.263	.402	.549	.711	.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	.130	.262	.399	.546	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	.129	.261	.398	.543	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	.129	.260	.396	.540	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	.128	.259	.395	.539	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	.128	.259	.394	.538	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	.128	.258	.393	.537	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	.128	.258	.393	.536	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	.128	.258	.392	.535	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	.128	.257	.392	.534	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	.127	.257	.392	.534	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	.127	.257	.391	.533	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	.127	.257	.391	.533	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	.127	.257	.391	.532	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	.127	.256	.390	.532	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	.127	.256	.390	.531	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	.127	.256	.390	.531	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	.127	.256	.390	.531	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	.127	.256	.389	.531	.684	.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	.127	.256	.389	.530	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	.127	.256	.389	.530	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	.127	.256	.389	.530	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	.126	.255	.388	.529	.681	.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	.126	.254	.387	.527	.679	.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	.126	.254	.386	.526	.677	.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	.126	.253	.385	.524	.674	.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291